An exploration of threefold bases in F-theory

1510.04978 & upcoming work with W. Taylor

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F-theory landscape program

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F-theory compactification on an elliptic CY4 M, with complex threefold base B.

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Our goal: explore large sets of (compact, smooth) bases; Characterize, Classify, Count.

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Another property: the number of complex structure moduli $h^{3,1}$ of the elliptic CY4 is maximal.

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- Almost done: (Morrison, Taylor 12'; Martini, Taylor 14'; Taylor, YNW 15')

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- We allow terminal singularity on elliptic CY4, which may correspond to neutral chiral matter in the 4D supergravity(Arras, Grassi, Weigand 16').

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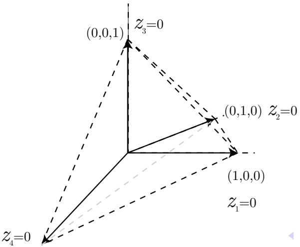
Description: a fan in the lattice $\mathbb{Z}^3$: Σ with set of 3D, 2D, 1D cones.

- 1D ray: v_i corresponds to divisor D_i; $z_i = 0$.

$N(v_i) = h^{1,1}(B) + 3$.

- 2D cone: $v_i v_j$ corresponds to curve $z_i = z_j = 0$.
- 3D cone: $v_i v_j v_k$ corresponds to point $z_i = z_j = z_k = 0$.



Toric threefolds

Generators of holomorphic section m_p of line bundle $L = \sum_i a_i D_i \Leftrightarrow$ points p in the dual lattice $\mathbb{Z}^3$:

$$\{p \in \mathbb{Z}^3, \forall v_i, \langle p, v_i \rangle \geq -a_i\}. \tag{2}$$

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Anti-canonical bundle $-K_B = \sum_i D_i$. Hence f and g are linear combinations of monomials in set $\mathcal{F}$ and $\mathcal{G}$:

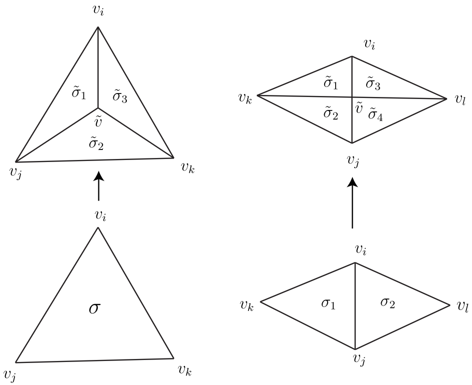
$$\mathcal{F} = \{p \in \mathbb{Z}^3, \forall v_i, \langle p, v_i \rangle \geq -4\}. \tag{4}$$

$$\mathcal{G} = \{p \in \mathbb{Z}^3, \forall v_i, \langle p, v_i \rangle \geq -6\}. \tag{5}$$

Blow up/down toric threefolds

(1) Blow up a point $v_i v_j v_k$: add another ray $\tilde{v} = v_i + v_j + v_k$.

(2) Blow up a curve $v_i v_j$: add another ray $\tilde{v} = v_i + v_j$.



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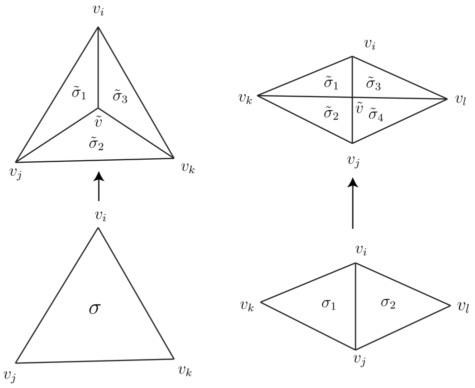

- The set $\mathcal{F}\&\mathcal{G}$ after the blow up is a subset of the previous ones.
- Blow up (4,6) curve does not change the set $\mathcal{F}\&\mathcal{G}$.

Random walk on the toric threefold landscape

- Start from $\mathbb{P}^3$, do a random sequence of 100,000 blow up/downs.
- Never pass through bases with cod-1 or cod-2 (4,6) singularities (excluding E_8 gauge group).
- In total 100 runs. $h^{1,1}(B) = 1 \sim 120$.

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SU(2)	SU(3)	G_2	SO(7)
13.6	2.0	9.7	4×10^{-6}
SO(8)	F_4	E_6	E_7
1.0	2.8	0.3	0.2

Average number of non-Higgsable gauge group on a base.

- 76% of bases have $SU(3) \times SU(2)$ non-Higgsable cluster.

Estimation of the number of distinct bases

- Do a limited random walk with cap $h^{1,1}(B) \leq 7$, get the number of bases $N(7)$ and $N(2)$.
- We know there is 1 base with $h^{1,1}(B) = 1$: $\mathbb{P}^3$. 27 bases with $h^{1,1}(B) = 27$.
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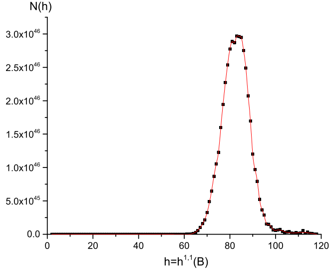

Total number $\sim 10^{48}$.

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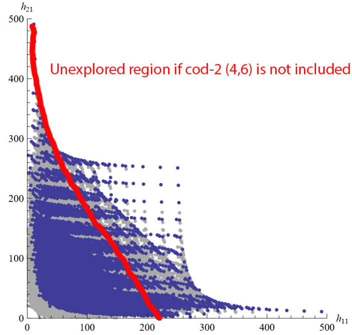
For the case of surface base/elliptic CY3, this exclude a large region.



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- Assign weight factor to each base on each sequence to compute the total number of resolvable/good bases with each $h^{1,1}(B)$.

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(3) We can consider more general starting point bases with non-Higgsable clusters.

New results

- The distribution of resolvable bases centralized at very large $h^{1,1} \sim 4,000$.
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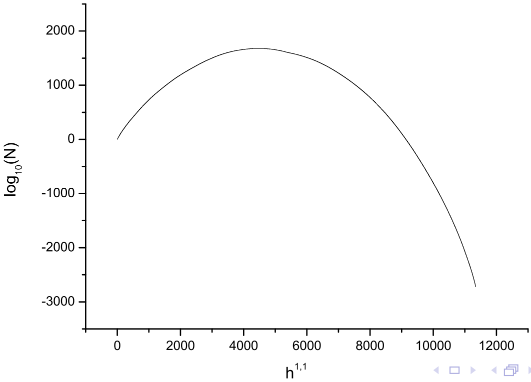

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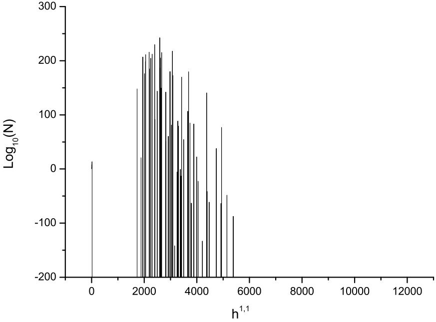

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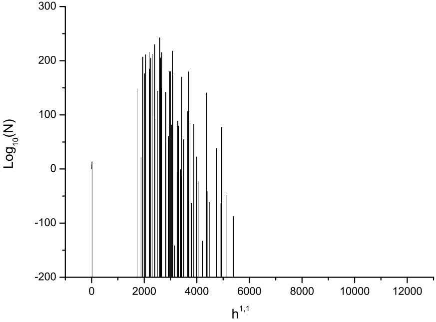

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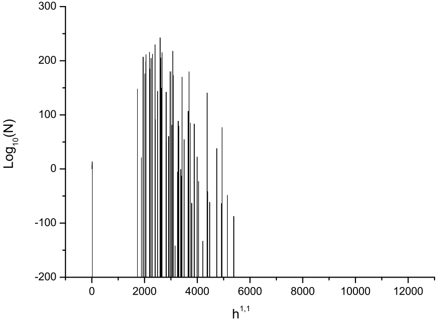

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- Autocracy. Similar story happens in the flux vacua story (YNW, Taylor 15'), where one geometry with $10^{272,000}$ flux vacua dominates.

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• The end points are not random, but they are not related by flop either. Give rise to the same CY4?

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