Moduli of Heterotic Backgrounds and their Effective Theories

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Based on work in collaboration with

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Introduction

Motivation and Overview

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- Higher curvature corrections induce torsional (non Ricci-flat) geometries.
- Harder to understand geometries. Loose toolbox of algebraic geometry and Kähler geometry.
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This talk: Heterotic string on manifolds with reduced structure group $SU(3)$ (and G_2 manifolds).

- Finite deformations of $SU(3)$ system, the superpotential and "effective theory" governing heterotic DGLA.
- (Short) review of infinitesimal (massless) moduli of $SU(3)$ compactifications.
- Higher order deformations and obstructions.
- (Infinitesimal moduli of heterotic string on G_2-structure Manifolds.)

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$$T\mathcal{M} = H^1_{\mathcal{D}}(\mathcal{Q}) \; .$$

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- *Step 2:* Understand geometry of $\mathcal{M}$ (Kähler metric, etc). Higher order deformations, obstructions (Yukawa couplings). Maurer-Cartan elements,

$$\mathcal{D}\mathcal{X} + \tfrac{1}{2}[\mathcal{X}, \mathcal{X}] = 0 \ ,$$

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- *Step 3:* Understand *quantum cohomology ring*. Include non perturbative effects such as world-sheet instantons, and quantum corrections (higher genus effects). Construct topological theory of corresponding structures?

The Heterotic $SU(3)$-system

$$\mathcal{M}_{10} = \mathcal{M}_4 \times X_6$$

The Heterotic Superpotential

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This theory has a superpotential given by [Becker et al 03, Cardoso et al 03, Lukas et al 05, McOrist 16, ..]

$$W = \int_X (H + i\mathrm{d}\omega) \wedge \Omega \,,$$

where the flux is given by

$$H = \mathrm{d}B + \frac{\alpha'}{4}\omega_{CS}(A) \,.$$

Note that the flux H is *gauge invariant* which imposes a gauge transformation on B through the Green-Schwarz mechanism.

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Supersymmetric (Minkowski) solutions require that:

$$\delta W = W = 0 \, ,$$

at the solution.

The Supersymmetry Conditions

From the superpotential we derive the following conditions on the $SU(3)$ structure [Strominger, Hull 86]

- d$\Omega = 0 \;\Rightarrow\; (X, J)$ is a *complex manifold*. Equivalent to $\overline{\partial}^2 = 0$, where d $= \partial + \overline{\partial}$ and
$$\overline{\partial} \;:\quad \Omega^{(p,q)}(X) \to \Omega^{(p,q+1)}(X)\,.$$
The operator $\overline{\partial}$ defines *Dolbeault cohomologies* $H^{(p,q)}_{\overline{\partial}}(X)$.
- $F \wedge \Omega = 0 \;\Leftrightarrow\; F^{(0,2)} = \overline{\partial}_A^2 = 0$, where $\overline{\partial}_A =$ d$_A{}^{(0,1)}$. We say the bundle (V, A) is *holomorphic*.
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There are also “D-term” conditions:

$$\mathrm{d}\left(e^{-2\phi}\omega \wedge \omega\right) = 0\,, \quad \omega \wedge \omega \wedge F = 0\,,$$

X is conformally balanced and A is Yang-Mills. This talk: Will assume *stable* bundles. D-terms not very relevant for moduli problem.

A Heterotic Effective Theory

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Clues from physics:

Know that superpotential is *holomorphic* $\Rightarrow$ A *finite* and *holomorphic* deformation of the heterotic $SU(3)$-system can be represented as a $(0,1)$-form

$$y = (x, \alpha, \mu) \in \Omega^{(0,1)}(Q_2)\ , \quad Q_2 = T^{*(1,0)}X \oplus \text{End}(V) \oplus T^{(1,0)}X\ ,$$

where $\mu \in \Omega^{(0,1)}(T^{(1,0)}X)$, $\alpha \in \Omega^{(0,1)}(\text{End}(V))$ and $x \in \Omega^{(0,1)}(T^{*(1,0)}X)$.

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A generic holomorphic deformation of the superpotential gives

$$\Delta W = \int_X \left(\langle y, \overline{\partial}_2 y\rangle + \tfrac{1}{3}\langle y, [y,y]\rangle + \mu^a \partial_a b\right) \wedge \Omega\,,$$

where for $y_1, y_2 \ \in\ \Omega^{(0,*)}(Q_2)$, $\langle y_1, y_2\rangle = \mu_1^a x_{2\,a} + \mu_2^a x_{1\,a} + \text{tr}\,(\alpha_1\alpha_2)$, $b \in \Omega^{(0,2)}(X)$ is auxillary, and

$$[\,,\,]\ :\ \ \Omega^{(0,p)}(Q_2) \times \Omega^{(0,q)}(Q_2) \to \Omega^{(0,p+q)}(Q_2)$$

satisfies Leibniz rule w.r.t. $\overline{\partial}_2$, and Jacobi identity modulo ∂_a-exact terms.

Maurer-Cartan Equation

The field b is an auxiliary filed and part of the deformation of B. Integrating it out gives the condition

$$\partial\Omega(\mu) = 0\,, \quad \Omega(\mu) = \tfrac{1}{2}\Omega_{abc}\mu^a\,\mathsf{d}z^{bc}\,,$$

and the “effective action” now reads

$$\Delta W = \int_X \left(\langle y, \overline{\partial}_2 y\rangle + \tfrac{1}{3}\langle y, [y,y]\rangle\right) \wedge \Omega\,,$$

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The action is invariant under the symmetries

$$y \to y + \overline{\partial}_2 c + [y,c] + \partial_a d\,, \quad c \in \Omega^0(Q_2)\,, \quad d \in \Omega^{(0,1)}(X)\,.$$

These symmetry transformations can be identified with diffeomorphisms, gauge transformations and B-field transformations of the background fields.

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Equation of Motion: $\quad \overline{\partial}_2 y + \tfrac{1}{2}[y, y] = \partial_a$-exact .

The MC-equation for the DGLA $\mathcal{A} = \left(\Omega^{(0,*)}\left(Q_2/\{\partial_a\text{-exact}\}\right), \overline{\partial}_2\,, [\,,\,]\right)$.

Infinitesimal Moduli and Massless Fields

Note: Will assume "technical assumption" such as the $\partial\bar{\partial}$-lemma, or $h^{(0,1)} = 0$.

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The infinitesimal (massless) moduli y satisfy the equation

$$\overline{\partial}_2 y = \partial_a\text{-exact} \ ,$$

where $\overline{\partial}_2 = \overline{\partial}_1 + \mathcal{H}$ defines Q_2 as an extension [Anderson et al 14]

$$[\mathcal{H}] \in \mathrm{Ext}^1(Q_1, T^{*(1,0)}X) = H^{(0,1)}_{\overline{\partial}_1}(Q_1^*, T^{*(1,0)}X) \ .$$

Here Q_1 is the extension of Atiyah defined by $\overline{\partial}_1 = \overline{\partial} + \mathcal{F}$ (curvature)

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Note: $\overline{\partial}_2^2 = 0 \Leftrightarrow (X, \Omega, \omega)$ is complex with a holomorphic vector bundle $(\text{End}(V), A)$ and satisfies heterotic Bianchi Identity. Applying "technical assumption", and modding out by symmetries one finds

$$[y] \in H^{(0,1)}_{\overline{\partial}_2}(Q_2) = H^{(0,1)}_{\overline{\partial}}(T^{*(1,0)}X) \oplus \ker(\mathcal{H})$$

$$\ker(\mathcal{H}) \subseteq H^{(0,1)}_{\overline{\partial}_1}(Q_1) = H^{(0,1)}_{\overline{\partial}}(\text{End}(V)) \oplus \ker(\mathcal{F}) ,$$

as was shown in [Anderson et al 14, delaOssa-EES 14, Garcia-Fernandez 15].

Maurer-Cartan Elements and Obstructions

The classically unobstructed fields y satisfy the Maurer-Cartan equation

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The moduli satisfying this equation correspond to free fields without couplings in the effective 4d Lagrangian.

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Hard to analyse the obstructions in general, but can make observations for special cases such as for “Calabi-Yau” compactifications.

Theorem [Anderson-Gray-Lukas-Ovrut 11]: The number of unobstructed moduli of the Atiyah algebroid Q_1 on a Calabi-Yau is bounded from below by $h^{(0,1)}(T^{(1,0)}X)$.

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Including α' effects and the heterotic anomaly cancellation condition (Bianchi Identity), this can be extended to

Theorem [delaOssa-Hardy-EES 15]: The number of unobstructed moduli of the heterotic algebroid Q_2 on a "Calabi-Yau" is bounded from below by the number of *massless* geometric moduli of the base X.

Conclusions and Outlook

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- Heterotic compactifications give a fertile ground for phenomenology, but the moduli problem is hard.
- From the heterotic superpotential we have derived an effective theory whose equation of motion naturally gives the MC equation of the heterotic DGLA.
- We have (re-)derived the infinitesimal moduli of the heterotic $SU(3)$-system, and considered higher order deformations and obstructions.

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Outlook, and work in progress:

- Apply to explicit examples to connect with phenomenology, other areas of string theory (AdS/CFT?), and differential geometry?
- What about non-perturbative effects, world sheet instantons, NS5-branes? Correct the Bianchi Identity and spoils holomorphic structure $\overline{\partial}_2^2 \neq 0$.
- Quantise quasi-topological action ΔW? Is there a corresponding topological world-sheet theory (e.g. AKSZ model or holomorphic beta-gamma system)? Compute invariants for heterotic geometries generalising DT-invariants and GW-invariants (relate to threshold corrections?). In progress with A. Ashmore, X. de la Ossa R. Minasian, C. Strickland-Constable.

Thank you!

Thank you for your attention, and happy 4th of July!

Compactifications to 3d

$$\mathcal{M}_{10} = \mathcal{M}_3 \times Y_7$$

G_2-structure compactifications

Supersymmetry requires the internal geometry Y to have a G_2-structure. Tangent bundle has a reduced structure group $G_2 \subset SO(7)$.

G_2 structure determined by a non-degenerate G_2 invariant three-form φ, where $\psi = *\varphi$ and the Hodge dual is given by the metric defined by φ.

Torsion classes (decomposed into irreduclible representations of G_2)

$$\mathrm{d}\varphi = \tau_1 \psi + 3\,\tau_7 \wedge \varphi + *\tau_{27}$$
$$\mathrm{d}\psi = 4\,\tau_7 \wedge \psi + *\tau_{14}\ .$$

Note that $\mathrm{d}\varphi = \mathrm{d}\phi = 0 \;\Leftrightarrow\; \nabla^{LC}\varphi = 0$ (G_2 *holonomy*).

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Supersymmetry requires [Papadopoulos et al 05, Lukas et al 10, Gray et al 12, ..]

$$\tau_1 = \mathrm{dvol}_{\mathcal{M}_3} \lrcorner H\ , \quad \tau_7 = \tfrac{1}{2}\mathrm{d}\phi$$
$$\tau_{14} = 0\ , \quad \tau_{27} = -H + \tfrac{1}{6}\tau_1\varphi - \tau_7 \lrcorner \psi\ .$$

Note, this is an *integrable* G_2-structure.

Gauge bundle: $F \wedge \psi = 0 \quad \Leftrightarrow \quad$ Curvature is an instanton.

The Canonical G_2 Complex

For integrable G_2 structure (in particular G_2 holonomy) we have the complex [Reyes-Carrion 93, Fernandez et al 98]

$$0 \to \Omega^0_1 \xrightarrow{\check{\mathrm{d}}} \Omega^1_7 \xrightarrow{\check{\mathrm{d}}} \Omega^2_7 \xrightarrow{\check{\mathrm{d}}} \Omega^3_1 \to 0\,.$$

Here $\check{\mathrm{d}} = \pi \circ$ d, where π is the projection onto the appropriate G_2 representation.

This is a differential complex, i.e. $\check{\mathrm{d}}^2 = 0$, iff the G_2 structure is integrable (in close analogy with integrable complex structure $\Leftrightarrow$ $\overline{\partial}^2 = 0$).

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This complex generalises to bundles (V, A)

$$0 \to \Omega^0_1(V) \xrightarrow{\check{\mathrm{d}}_A} \Omega^1_7(V) \xrightarrow{\check{\mathrm{d}}_A} \Omega^2_7(V) \xrightarrow{\check{\mathrm{d}}_A} \Omega^3_1(V) \to 0 \, .$$

where $\check{\mathrm{d}}_A = \pi \circ \mathrm{d}_A$. This is a differential complex iff $F \wedge \psi = 0$.

These complexes are elliptic [Reyes-Carrion 93] $\Rightarrow$ the corresponding cohomologies $\check{H}^*(V)$ are finite dimensional.

The heterotic G_2 system naturally gives rise to a differential

$$\check{\mathcal{D}} = \begin{pmatrix} \check{\mathrm{d}}_A & \check{\mathcal{F}} \\ \check{\mathcal{F}} & \check{\mathrm{d}}_\theta \end{pmatrix} \; : \; \check{\Omega}^p \begin{pmatrix} \mathrm{End}(V) \\ TY \end{pmatrix} \;\rightarrow\; \check{\Omega}^{p+1} \begin{pmatrix} \mathrm{End}(V) \\ TY \end{pmatrix} ,$$

where the map(s) $\check{\mathcal{F}}$ are given by

$$\Delta \in \Omega^p(TY) \; : \; \check{\mathcal{F}}(\Delta) = \pi\left(F_{mn}\mathrm{d}x^n \wedge \Delta^m\right)$$

$$\alpha \in \Omega^p(\mathrm{End}(V)) \; : \; \check{\mathcal{F}}(\alpha)^m = \frac{\alpha'}{4}\,\pi\left[\mathrm{tr}\left(g^{mn}F_{nq}\mathrm{d}x^q \wedge \alpha\right)\right] .$$

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and π denotes the appropriate projection.

The connection d_θ has connection symbols

$$\theta_{mn}{}^p = \Gamma_{nm}{}^p ,$$

where Γ denotes the unique metric connection with totally anti-symmetric torsion preserving the G_2 structure, $\nabla^\Gamma \varphi = 0$.

Using the heterotic supersymmetry conditions, it can be shown that $\check{\mathcal{D}}^2 = 0$.

Theorem (delaOssa-Larfors-EES) The infinitesimal moduli space of the heterotic G_2 system is

$$T\mathcal{M} = \check{H}^1_{\check{\mathcal{D}}}(\mathcal{Q}_1)\ .$$

Scetch of proof: Identify infinitesimal deformations with $\check{\mathcal{D}}$-closed forms. Identify heterotic symmetries (diffeomorphisms and gauge-transformations, B-field transformations) with $\check{\mathcal{D}}$-exact forms.

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Note when $\alpha' = 0$ the differential reads

$$\check{\mathcal{D}} = \begin{pmatrix} \check{\mathrm{d}}_A & \check{\mathcal{F}} \\ 0 & \check{\mathrm{d}}_\theta \end{pmatrix} \ :\ \check{\Omega}^p \begin{pmatrix} \text{End}(V) \\ TY \end{pmatrix} \ \rightarrow\ \check{\Omega}^{p+1} \begin{pmatrix} \text{End}(V) \\ TY \end{pmatrix} .$$

Get short exact extension of complexes, $\mathcal{Q}_1 = \text{End}(V) \oplus TY$

$$0 \rightarrow \check{\Omega}^*(\text{End}(V)) \rightarrow \check{\Omega}^*(\mathcal{Q}_1) \rightarrow \check{\Omega}^*(TY) \rightarrow 0\ .$$

Long exact sequence in cohomology gives

$$T\mathcal{M}_1 = \check{H}^1(\mathcal{Q}_1) = \check{H}^1(\text{End}(V)) \oplus \ker(\check{\mathcal{F}})\ , \quad \ker(\check{\mathcal{F}}) \subseteq \check{H}^1(TY)\ ,$$

where $\check{H}^1(TY)$ contains both metric and B-field deformations.

Differences with $SU(3)$-System

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- The system is more compact (not an extension).
- No separation of structures, $\check{\mathcal{D}}$ has everything. Opposite direction holds within perturbative curvature expansion: $\check{\mathcal{D}}^2 = 0$ implies most of the heterotic G_2-system, except that Lee form τ_7 is exact (given by the dilaton).
- Embedding $SU(3)$-system in G_2-system puts holomorphic and hermitian structures on equal footing.
- The differential $\check{\mathcal{D}}$ makes it clear that the heterotic $SU(3)$-system can be viewed as a holomorphic Yang-Mills connection on $\mathcal{Q}$.
- No "technical assumption".

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- No separation of structures, $\check{\mathcal{D}}$ has everything. Opposite direction holds within perturbative curvature expansion: $\check{\mathcal{D}}^2 = 0$ implies most of the heterotic G_2-system, except that Lee form τ_7 is exact (given by the dilaton).
- Embedding $SU(3)$-system in G_2-system puts holomorphic and hermitian structures on equal footing.
- The differential $\check{\mathcal{D}}$ makes it clear that the heterotic $SU(3)$-system can be viewed as a holomorphic Yang-Mills connection on $\mathcal{Q}$.
- No “technical assumption”.

The disadvantages are:

- The system is more compact (not an extension) $\Rightarrow$ harder to identify usual cohomologies within $H^1_{\check{\mathcal{D}}}(\mathcal{Q}_1)$.
- Harder to compute $H^1_{\check{\mathcal{D}}}(\mathcal{Q}_1)$. No clear notion of Poincare lemma or sheaf cohomology $\Rightarrow$ hard to get to by algebraic methods. Homogeneous examples?