Constraining the Moduli Space of Trees in the F-theory Landscape

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Moduli Space of F-theory Compactifications

- The landscape of F-theory compactifications is not well-understood
- Moduli problems are ubiquitous in both mathematics and physics. (How do we classify and parametrize a family of geometric objects?)
- The solution to a moduli problem may require an arbitrarily general global structure (e.g. schemes, stacks, derived stacks)
- A common practice is to study compactifications of moduli spaces and restrict to studying "nicer" objects (GIT, stability conditions, etc.)
- 4D F-theory vacua $\subset \left\{ \begin{array}{l} \text{moduli space of elliptic fibrations of} \\ \text{Calabi-Yau fourfolds} \end{array} \right\}$

Question

How do we restrict our full moduli space of elliptically fibered Calabi-Yau fourfolds to those that give consistent F-theory compactifications (amenable to a systematic analysis of non-Higgsable clusters)?

Classification of F-theory Geometries

- Let $\pi : X \to B$ be an elliptically fibered Calabi-Yau fourfold
- Study by classifying algebraic threefold bases B.
- Obstruction: threefold MMP much more complicated
- Starting point: assume π has section and B smooth and toric
- Weierstrass form

$$y^2 = x^3 + fx + g$$

  with $f \in \Gamma(\mathcal{O}_B(-4K_B))$, $g \in \Gamma(\mathcal{O}_B(-6K_B))$.
- Weak Fano bases do not give rise to NHCs (smooth elliptic fibration)
- Build geometries by taking the induced fibration over blowups

Problem

How do we constrain blowups to yield well-behaved physics?

Hayakawa-Wang Constraint

- To deform to a low-energy supergravity description, need singular total space to be at finite distance from a Calabi-Yau manifold

Theorem (Wang)

Let X be a Calabi-Yau variety which admits a smoothing to a Calabi-Yau manifold. If X has only canonical singularities, then X has finite Weil-Petersson distance along any such smoothing.

- Basic Idea behind hypothesis on singularities:
  1. Canonical singularities $\Rightarrow H^{n,0}(\tilde{X}, \mathbb{C}) \neq 0$ for any resolution $\tilde{X}$
  2. Central fiber of semistable degeneration of X has $h^{n,0} = 1$ component
  3. $h^{n,0} = 1$ component $\iff$ finite Weil-Petersson distance (involved)
- For X a Weierstrass model over B, this is equivalent to the condition that there are no (4,6) divisors in B, i.e. $ord_D(f) < 4$ or $ord_D(g) < 6$

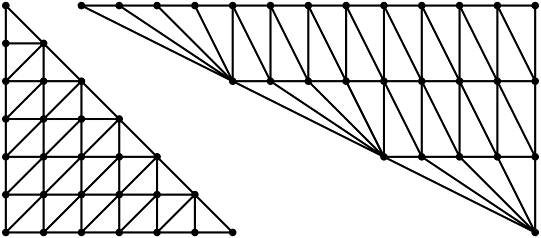
Constraints on Trees

- For B toric, this condition is an easy combinatorial computation
- Recall construction (Jim's talk):
  1. Begin with weak Fano toric threefold (triangulation of reflexive Δ°)
  2. Apply smooth T-invariant blowups (rays of fan are $\sum_i a_i v_i$, $v_i \in \Delta^\circ$)
- No (4,6) divisors $\iff$ for all $v \in Fan(B)$, there exists $m \in a\Delta$ s.t. $\langle m, v \rangle < 0$ for $a = 4, 6$ (reflexivity and definition of $\Gamma(\mathcal{O}(-aK_B))$)
- A sufficient condition is for all rays v to satisfy the property $\sum_i a_i < 6$
- Recursive procedure given by the following diagram

$$\begin{array}{ccccc} X' & \longrightarrow & \tilde{X} & \longrightarrow & X \\ & & \downarrow & \lrcorner & \downarrow \\ & & B' & \longrightarrow & B \end{array}$$

Constraints on Trees

- Vanishing multiplicities along higher codimension subvarieties
  1. Curves: $m_C \geq (4,6) \Rightarrow$ crepant blowup, $m_C < (8,12) \Rightarrow$ no (4,6)
  2. Points: $m_C \geq (8,12) \Rightarrow$ crepant blowup, $m_C < (12,16) \Rightarrow$ no (4,6)
  3. (4,6) curves can lead to non-flat fibration and CFT in low energy theory
  4. Not issues in our ensemble!



- Uniqueness of geometries: Geometries in two largest ensembles from two largest faces not isomorphic, leaves cannot map to different heights (follows easily by linearity of toric morphisms on fans)

Conclusion

- Top-down: Hayakawa-Wang restricts moduli space of elliptically fibered Calabi-Yau varieties to consistent F-theory compactifications
- Bottom-Up: systematically build $\approx 10^{755}$ non-isomorphic smooth toric threefolds satisfying (4,6) and demonstrating universality of NHCs
- Future Work:
  1. The moduli space of F-theory compactifications may be disconnected (Oda's factorization conjecture)
  2. Systematic analysis may be done on more general Kähler varieties which are locally toric orbifolds (see Cody's talk)