T-branes and Stability Walls

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Based on: *Marchesano, Savelli, S.S. '17*
See also talk by F. Marchesano

SPLE Advanced Grant

BPS-stability of 7-branes

- Worldvolume theory of stack of 7-branes on 4-cycle S is 8d $\mathcal{N} = 1$ SYM
- Bosonic field content:
  $\Phi \in H^0(S, \mathrm{End}(V) \otimes K_S) \cong H^{2,0}(S, \mathrm{End}(V))$
  $A \in H^1(S, \mathrm{End}(V))$
- Vacuum is stable if BPS-conditions are satisfied

BPS-stability of 7-branes

- They split into F-term conditions $F^{2,0} = 0$ and $\overline{\partial}_A \Phi = 0$ and D-term conditions $J \wedge F + \frac{1}{2}[\Phi, \Phi^\dagger] = 0$
- F-term conditions are holomorphic data, i.e. they are invariant under complexified gauge transformations of the vector bundle
- D-terms depend on Kähler moduli and are subjected to α'-corrections *Marchesano and S.S. 2016*

Intersecting branes

- Cartan of Φ parametrises transversal deformations of brane stack
- consider 2 coinciding branes, i.e. gauge group $U(2)$
- Switching on a vev $\Phi = \begin{pmatrix} v & \\ & -v \end{pmatrix}$ breaks the gauge group to $U(1)^2$
- geometrically we are taking apart the branes
- if v is non-constant the symmetry is restored along its vanishing loci — intuitively this describes a pair of branes intersecting along the curve $v = 0$

T-branes

- What happens if we switch on a non-Cartan vev? *Cecotti, Cordova, Heckman, and Vafa 2011*
- $\Phi = \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix}$ breaks the gauge group but does not encode geometric deformation
  (different context *Vafa and Witten 1994*)
- information is lost completely in standard M-/F-theory uplift but well-defined states from world-volume perspective
- Some interest in the last years *Anderson, Heckman, and Katz 2014; Collinucci and Savelli 2014; Collinucci and Savelli 2015; Collinucci, Giacomelli, Savelli, and Valandro 2016; Anderson, Heckman, Katz, and Schaposnik 2017; Collinucci, Giacomelli, and Valandro 2017*

T-branes

- Most work has been focussed on analysis on locally flat environments
- Consider T-branes on a compact, simply connected 4-cycle S
- Assume for now that Φ has no poles
- Solutions are unique and require non-harmonic flux away from Kähler-Einstein
- There is no stable T-branes on positive curvature 4-cycles or K3
  $\rightarrow$ *See F. Marchesano's talk*
- Here: Role of Kähler moduli

Coinciding Branes

- Consider the case of vanishing Higgs-vev $< \Phi >= 0$ of a gauge theory $V \equiv \mathcal{L} \oplus \mathcal{L}^{-1}$
- study fluctuations around this vacuum
  $\delta\Phi = \begin{pmatrix} v & m \\ p & -v \end{pmatrix}$ and $\delta A = \begin{pmatrix} 0 & a_+ \\ a_- & 0 \end{pmatrix}$
- transforming as
  $m \in H^0(S, \mathcal{M})$ $\quad p \in H^0(S, \mathcal{P})$ $\quad v \in H^0(S, K_S)$
  $a_+ \in H^1(S, \mathcal{P})$ $\quad a_- \in H^1(S, \mathcal{M})$
  $\mathcal{M} \equiv \mathcal{L}^2 \otimes K_S$ and $\mathcal{P} \equiv \mathcal{L}^{-2} \otimes K_S$

T-branes from Coinciding Branes

- We are interested in T-branes $\Rightarrow$ focus for now on $v \equiv 0$
- after dimensional reduction to 4d, the D-term condition becomes
  $\sum_m |m|^2 + \sum_{a_+} |a_+|^2 - \sum_p |p|^2 - \sum_{a_-} |a_-|^2 = \xi$
- depending on the Fayet-Iliopoulos term
  $\xi = -2 \int_S J \wedge c_1(\mathcal{L})$
- Assume we are in a configuration with m-type zero modes and we may have $\xi > 0$.
  $\rightarrow$ Are wall-crossing phenomena possible? I.e. is it possible for the T-brane to decay to a non-supersymmetric state on the side $\xi < 0$?

Walls of Stability

- this is the case if there are no p or a_- modes
  $\Rightarrow$ If $H^0(S, \mathcal{P}) = H^1(S, \mathcal{M}) = 0$ no solution across the wall
  $\Rightarrow$ T-brane decays into non-mutually supersymm. 7-brane components
- From the index we may derive a condition for decay
  $I = \#(+) - \#(-) = 2\int_S c_1(\mathcal{L}) \wedge c_1(K_S)$
- $I \leq 0 \Rightarrow$ No decay.
- $I > 0 \Rightarrow$ Decay possible.

The Hitchin-case

- Consider simplest case $V = K_S^{-\frac{1}{2}} \oplus K_S^{\frac{1}{2}} \Rightarrow \mathcal{M}$ is trivial bundle and $\mathcal{P} = K_S^2$
- $H^1(S, \mathcal{M})$ vanishes automatically because S is simply connected by assumption
- to satisfy the necessary condition, we need $I = -\int_S \mathrm{c}_1(K_S)^2 > 0$
- $\mathcal{P}$ has no sections iff $0 > \int_S J \wedge \mathrm{c}_1(\mathcal{P}) = 2 \int_S J \wedge \mathrm{c}_1(K_S) = 2\xi$
- For ξ to be able to change sign (i.e. for the "wall" to exist), S needs to have indefinite curvature

- repeat analysis for two intersecting branes on $S_{1/2}$ carrying line bundles $\mathcal{L}_{1/2}$ and intersecting along the curve $\mathcal{C}$
- no gauge-field d.o.f. and 4d D-terms given by $\sum_m |m|^2 - \sum_p |p|^2 = \int_{S_2} J \wedge F_2 - \int_{S_1} J \wedge F_1$
- where now $m \in H^0(\mathcal{C}, \mathcal{L}_2^{-1}|_\mathcal{C} \otimes \mathcal{L}_1|_\mathcal{C} \otimes K_\mathcal{C}^{1/2})$ and $p \in H^0(\mathcal{C}, \mathcal{L}_2|_\mathcal{C} \otimes \mathcal{L}_1^{-1}|_\mathcal{C} \otimes K_\mathcal{C}^{1/2})$
- relevant index is $I = \deg \mathcal{L}_1|_\mathcal{C} - \deg \mathcal{L}_2|_\mathcal{C}$

Intersecting Branes