The global gauge group structure of F-theory compactifications with U(1)s

based on arXiv:1706.08521 with Mirjam Cvetič

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Motivation

- Global gauge group structure of non-abelian symmetry given by Mordell–Weil torsion.
  [Apsinwall, Morrison '98], [Mayrhofer, Morrison, Till, Weigand '14]
- F-theory 'Standard Models' realize precisely physical spectrum under
  $\mathfrak{g}_{\text{SM}} = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)_Y$ [LL, Weigand '14, '16], [Klevers et al '14], [Cvetič et al '15]
  $\Longrightarrow$ gauge group $[SU(3) \times SU(2) \times U(1)_Y]/\mathbb{Z}_6$?
- What is the geometric origin?

Outline

Review: Shioda map and $\mathfrak{u}(1)$s in F-theory

- Sections σ_k of smooth elliptic Calabi–Yau $\pi : Y \to B$ form Mordell–Weil (MW) group: $\text{MW}(Y) = \mathbb{Z}^m \times \prod_t \mathbb{Z}_{k_t}$. $[\sigma_k] = S_k, k = 0, ..., m$ are independent integer divisors.
- Codim. 1 singular fibers $\Rightarrow$ E_i (Cartan divisors of non-ab. algebra $\mathfrak{g}$); Shioda–Tate–Wazir: $H^{1,1}(Y, \mathbb{Q}) = \langle S_1, ..., S_m \rangle \oplus \langle S_0, E_i \rangle \oplus \pi^* H^{1,1}(B, \mathbb{Q})$.
- $\exists$ injective homomorphism (Shioda map) $\varphi : \text{MW}(Y) \to H^{1,1}(Y, \mathbb{Q})$, s.t. image is 'orthogonal' to $\langle S_0, E_i \rangle \oplus H^{1,1}(B, \mathbb{Q})$.
- General form: $\varphi(\sigma) = \lambda\,(S - S_0 + \sum_i l_i\, E_i\, (+D_B))$, λ a priori not constrained; for now, fix $\lambda = 1$.
- $\mathfrak{u}(1)$ gauge field A arise from KK-reduction $C_3 = A \wedge \varphi(\sigma) + ...$

Fractional $\mathfrak{u}(1)$ charges

- $\mathfrak{u}(1)$ charge of matter states from codim. 2 fibral curves Γ given by $\Gamma \cdot \varphi(\sigma)$ with $\varphi(\sigma) = S - S_0 + \sum_i l_i \, E_i$.
- Coefficients l_i are determined by 'orthogonality' of $\varphi(\sigma)$ with $E_i = (\mathbb{P}^1_i \to \{\theta\})$; explicitly, $l_i = \sum_j (C^{-1})_{ij} \, (S - S_0) \cdot \mathbb{P}^1_j$, with $C_{ij} = -E_i \cdot \mathbb{P}^1_j$.

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- l_i are in general fractional, depend on the *fibre split type* and gauge algebra $\mathfrak{g}$. However, there is always an integer κ s.t. $\forall i : \kappa \, l_i \in \mathbb{Z}$.
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  $\Longrightarrow \; l_i \, \mathbf{v}_i = l_i \, (E_i \cdot \Gamma_v) = l_i \, (E_i \cdot \Gamma_w - \beta_k \, C_{ik}) = l_i \, \mathbf{w}_i - \underbrace{\beta_k \, (S - S_0) \cdot \mathbb{P}^1_k}_{\in \mathbb{Z}}$

Non-trivial central element from Shioda map

(a) There is $\kappa \in \mathbb{N}$ s.t. $\forall i: \kappa\, l_i \in \mathbb{Z} \Longrightarrow q_{\mathfrak{u}(1)} = \frac{n}{\kappa}, n \in \mathbb{Z}$ (pick smallest such κ).

(b) Two weights $\mathbf{w}, \mathbf{v}$ in the same $\mathfrak{g}$-rep $\mathcal{R}_\mathfrak{g}$: $l_i \mathbf{w}_i = l_i \mathbf{v}_i \mod \mathbb{Z} =: L(\mathcal{R}_\mathfrak{g})$ ($\Rightarrow \kappa\, L(\mathcal{R}_\mathfrak{g}) \in \mathbb{Z}$).

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$$\Longrightarrow G_{\text{global}} = \frac{U(1) \times G}{\langle C \rangle} \cong \frac{U(1) \times G}{\mathbb{Z}_\kappa}$$

Example: F-theory ‘Standard Model’

Toric construction with gauge algebra $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$. [Klevers et al ’14], [Cvetič et al ’15]

- $\varphi(\sigma) = S - S_0 + \frac{1}{2} E_1^{\mathfrak{su}(2)} + \frac{1}{3}(2\, E_1^{\mathfrak{su}(3)} + E_2^{\mathfrak{su}(3)}) \Rightarrow C^6 = 1$,
  so $G_{\text{global}} = [SU(3) \times SU(2) \times U(1)]/\langle C \rangle \cong [SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$.
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$\mathcal{R}_{\mathfrak{su}(3)\oplus\mathfrak{su}(2)}$	$(\mathbf{3}, \mathbf{2})$	$(\mathbf{3}, \mathbf{1})$	$(\mathbf{1}, \mathbf{2})$	$(\mathbf{1}, \mathbf{1})$
$L(\mathcal{R})$	$1/6$	$2/3$	$1/2$	0

geometrically realized matter: $(\mathbf{3}, \mathbf{2})_{1/6}\,, \quad (\mathbf{1}, \mathbf{2})_{-1/2}\,, \quad (\mathbf{3}, \mathbf{1})_{2/3}\,, \quad (\mathbf{3}, \mathbf{1})_{-1/3}\,, \quad (\mathbf{1}, \mathbf{1})_{1}$
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Similar situation in models with $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)_1 \oplus \mathfrak{u}(1)_2$ [LL, Weigand '14, '16].

After identifying hypercharge: $G_{\text{global}} = \dfrac{SU(3) \times SU(2) \times U(1)_Y \times U(1)_\perp}{\mathbb{Z}_6 \times \mathbb{Z}_\kappa}$

Global group structure as charge constraints

- σ torsional $\Rightarrow \varphi(\sigma) = 0$ (no $\mathfrak{u}(1)$), global group structure $G/\mathbb{Z}_\kappa$
  $\Longrightarrow$ not all $\mathfrak{g}$-reps allowed. [Mayrhofer, Morrison, Till, Weigand '14]
- σ free, then global group structure $[U(1) \times G]/\mathbb{Z}_\kappa \Rightarrow \mathfrak{u}(1)$ charges of $\mathfrak{g}$-reps constrained:
  For $\mathcal{R}^{(i)} = (q^{(i)}, \mathcal{R}_{\mathfrak{g}}^{(i)})$ we have $q^{(i)} = L(\mathcal{R}_{\mathfrak{g}}^{(i)}) \mod \mathbb{Z}$.
  For $\mathfrak{g} = \mathfrak{su}(5)$: [Braun, Grimm, Keitel '13], [Lawrie, Schäfer-Nameki, Wong, '15]

Argument derived with normalization $\lambda = 1$ for Shioda map $\Rightarrow$ 'preferred' charge normalization in F-theory: can read off global gauge group from fractional $\mathfrak{u}(1)$ charges.

Equivalently:
MW-group finitely generated $\longrightarrow$ global gauge group structure, refined charge quantization.

An F-theory 'swampland' criterion

- In preferred normalization, singlets ($E_i \cdot \Gamma = 0$) have integral $\mathfrak{u}(1)$ charges.
  Observation: in all $\mathfrak{u}(1)$-models with matter, smallest singlet charge is 1.
  $\Longrightarrow$ use singlets as 'measuring stick' for charges.
- Necessary condition for field theory to be F-theory compactification:
  normalize $\mathfrak{u}(1)$ such that all singlets charges are as above
  $\Rightarrow$ for all matter $\mathcal{R}^{(i)} = (q^{(i)}, \mathcal{R}_{\mathfrak{g}}^{(i)})$ we have $q^{(i)} - L(\mathcal{R}_{\mathfrak{g}}^{(i)}) \in \mathbb{Z}$. This implies:
  (1) If $\mathcal{R}^{(1)} = (q^{(1)}, \mathcal{R}_{\mathfrak{g}})$ and $\mathcal{R}^{(2)} = (q^{(2)}, \mathcal{R}_{\mathfrak{g}})$, then $q^{(1)} - q^{(2)} \in \mathbb{Z}$.
  (2) If $\bigotimes_{i=1}^{n} \mathcal{R}_{\mathfrak{g}}^{(i)} = \mathbf{1}_{\mathfrak{g}} \oplus \ldots$, then $\sum_{i=1}^{n} q^{(i)} \in \mathbb{Z}$.

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- Problem: What if no singlets? (Non-higgsable $\mathfrak{u}(1)$ in 6D without matter [(Martini), Morrison, (Park), Taylor ’14, ’16], [Wang ’17]) Situation in 4D?
- Other manifestations of the preferred normalization in field theory?

Conclusions & outlook

- Shioda map of sections $\Rightarrow$ non-trivial global gauge group structure of $\mathfrak{u}(1) \oplus \mathfrak{g}$ theory. Example: F-theory explicitly realizes Standard Model $[SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$.
- Equivalently: refined charge quantization condition. In 'preferred' charge normalization:
  1. Charge lattice of $\mathcal{R}_{\mathfrak{g}}$-matter integer spaced despite fractional charges.
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- Possible F-theory swampland criterion?
- What about charges of massive states? Spectrum of non-local operators?
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