Universality in a Large Ensemble of Geometries

Jim Halverson

Northeastern University

String Phenomenology 2017

Based on work
with Cody Long and Ben Sung.

Some Advertisements

- Cody Long's Talk: Weak Coupling Limits

- Ben Sung's Talk: Moduli, Morphisms, and Bounds

- Brent Nelson's Talk: Machine Learning

Q: how to approach large ensembles in the landscape?

Q: what constitutes ``large”?

Outline

- Geometrically Non-Higgsable Clusters
- Construction
- Universality

Non-Higgsable Clusters

7-brane Gauge Sectors

- CY elliptic fib. over B, extra spatial dimensions.

$$y^2 = x^3 + fx + g \qquad \text{Discrim: } \Delta = 0$$

- 7-branes on $\Delta = 0$, source $\tau = C_0 + e^{-\phi}$
- Seven-brane Gauge Sectors

F_i	l_i	m_i	n_i	Sing.	G_i
I_0	≥ 0	≥ 0	0	none	none
I_n	0	0	$n \geq 2$	A_{n-1}	$SU(n)$ or $Sp(\lfloor n/2 \rfloor)$
II	≥ 1	1	2	none	none
III	1	≥ 2	3	A_1	$SU(2)$
IV	≥ 2	2	4	A_2	$SU(3)$ or $SU(2)$
I_0^*	≥ 2	≥ 3	6	D_4	$SO(8)$ or $SO(7)$ or G_2
I_n^*	2	3	$n \geq 7$	D_{n-2}	$SO(2n-4)$ or $SO(2n-5)$
IV^*	≥ 3	4	8	E_6	E_6 or F_4
III^*	3	≥ 5	9	E_7	E_7
II^*	≥ 4	5	10	E_8	E_8

$$f = \tilde{f} \prod_i x_i^{l_i}$$

$$g = \tilde{g} \prod_i x_i^{m_i}$$

$$\Delta = \tilde{\Delta} \prod_i x_i^{\min(3l_i, 2m_i)} =: \tilde{\Delta} \prod_i x_i^{n_i}$$

Gauge Sectors for Generic CS

$$f = \tilde{f} \prod_i x_i^{l_i}$$

$$g = \tilde{g} \prod_i x_i^{m_i}$$

$$\Delta = \tilde{\Delta} \prod_i x_i^{\min(3l_i, 2m_i)} =: \tilde{\Delta} \prod_i x_i^{n_i}$$

F_i	l_i	m_i	n_i	Sing.	G_i
I_0	≥ 0	≥ 0	0	none	none
I_n	0	0	$n \geq 2$	A_{n-1}	$SU(n)$ or $Sp(\lfloor n/2 \rfloor)$
II	≥ 1	1	2	none	none
III	1	≥ 2	3	A_1	$SU(2)$
IV	≥ 2	2	4	A_2	$SU(3)$ or $SU(2)$
I_0^*	≥ 2	≥ 3	6	D_4	$SO(8)$ or $SO(7)$ or G_2
I_n^*	2	3	$n \geq 7$	D_{n-2}	$SO(2n-4)$ or $SO(2n-5)$
IV^*	≥ 3	4	8	E_6	E_6 or F_4
III^*	3	≥ 5	9	E_7	E_7
II^*	≥ 4	5	10	E_8	E_8

- f,g usually factorize, giving 7-branes can't split.
- Non-Higgsable 7-brane (NH7). Form clusters (NHC).

$$G \in \{E_8, E_7, E_6, F_4, SO(8), SO(7), G_2, SU(3), SU(2)\}$$

- 6d: either pure SYM, or not enough matter to Higgs.

NHC for Toric Bases

- Compute f,g-polytopes

$$\Delta_f = \{m \in \mathbb{Z}^3 \,|\, m \cdot v_i + 4 \geq 0 \; \forall i\} \qquad \Delta_g = \{m \in \mathbb{Z}^3 \,|\, m \cdot v_i + 6 \geq 0 \; \forall i\}$$

- Map to monomials

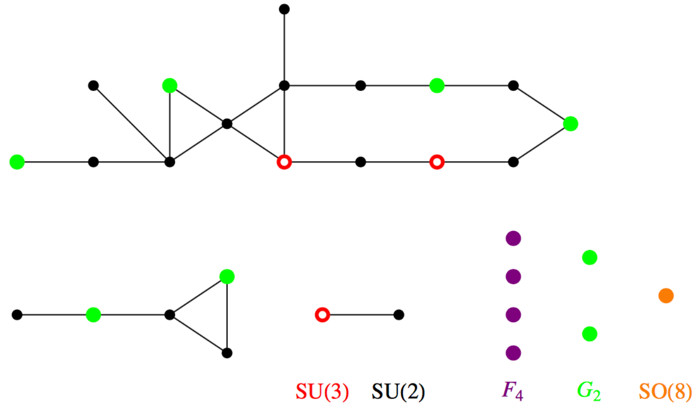
$$m_f \in \Delta_f \; \mapsto \; \prod_i x_i^{m_f \cdot v_i + 4} \qquad m_g \in \Delta_g \; \mapsto \; \prod_i x_i^{m_g \cdot v_i + 6}$$

- Construct most general f,g, look for factors.

$$f = \sum_{m_f \in \Delta_f} a_f \prod_i x_i^{m_f \cdot v_i + 4} \qquad g = \sum_{m_g \in \Delta_g} a_g \prod_i x_i^{m_g \cdot v_i + 6}$$

Some Selected Progress

- Morrison, Taylor: classifications in 6d.
- Morrison, Taylor: 4d paper, new features (loops e.g.)
- J.H., Taylor: P1 bundles over M-T torics.
  $> \ 10^5$ examples. 93% NHC.
- Grassi, J.H., Shaneson, Taylor: Some nice SM features.
- Taylor, Wang: Monte Carlo exploration, e.g.



Construction

[J.H., Long, Sung]

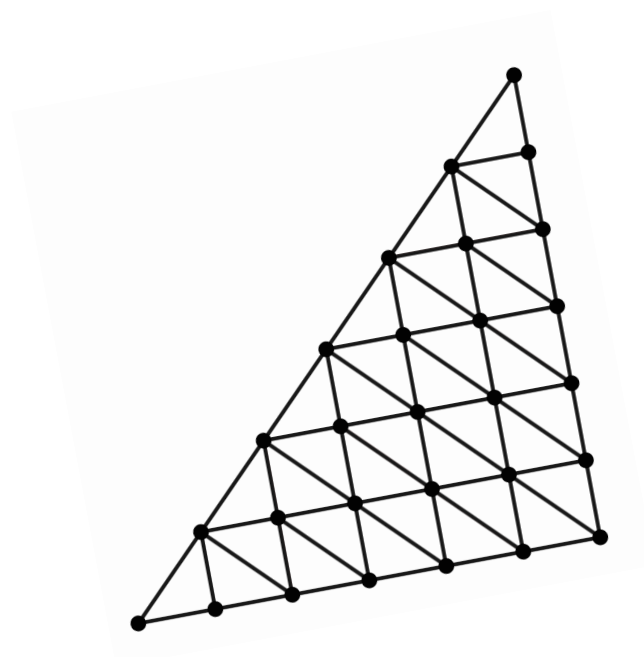
The Starting Point

- Extra dimensions: B_i smooth toric weak-Fano

- Determine by FRST of 3d reflexive polytope.

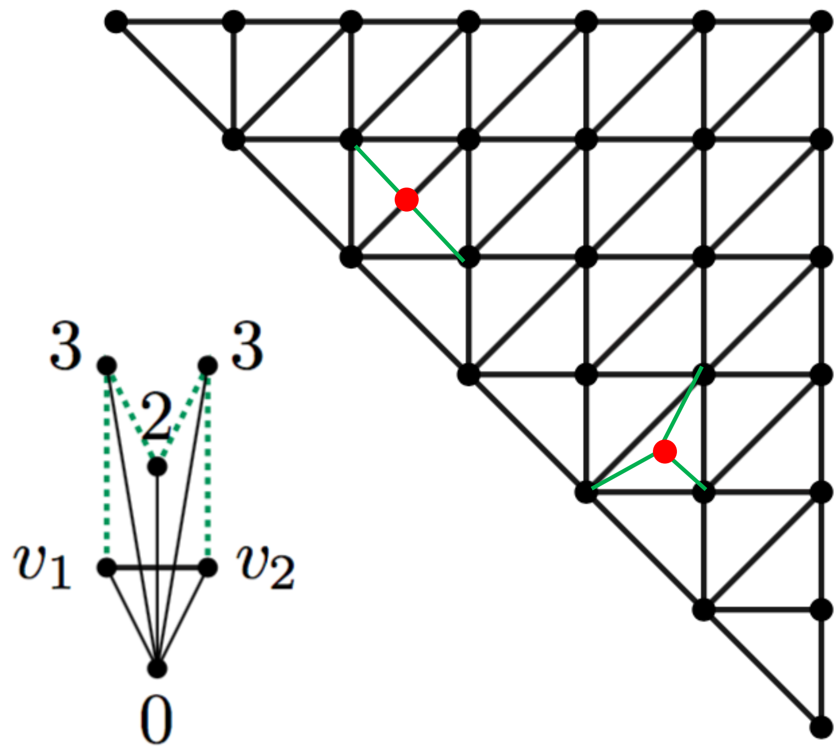
- $O(10^{15})$ such B_i. **[J.H., Tian]** **[Carifio, J.H., Krioukov, Nelson]**

- No NHC!

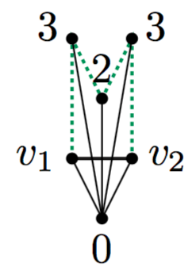


Visualizing Topological Transitions

- Curve blow: subdivide edge with vert v_1, v_2 by $v_e = v_1 + v_2$.
- Point blow: subdivide face with vertices v_1, v_2, v_3 by $v_e = v_1 + v_2 + v_3$.
- Iterate! Seq. of blow-ups.



Language, for Brevity

- ``Sequence of Blow-ups” = 

= Tree.

- Initial face or edge = patch or ground.

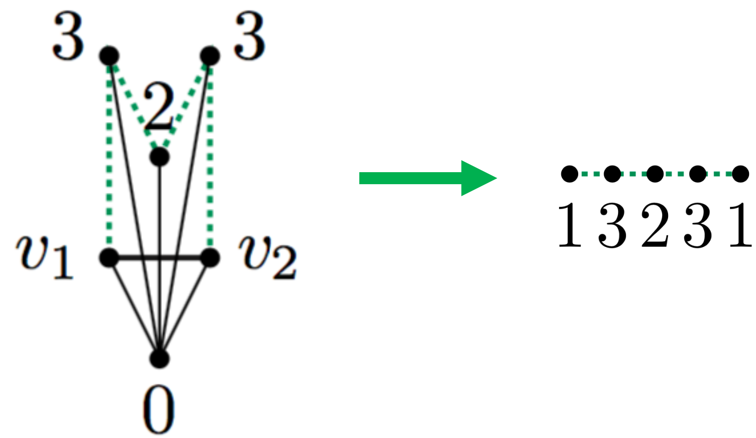
- v in tree = leaf. $v = a\, v_1 + b\, v_2 + c\, v_3$

- a + b + c = height of leaf. See above.

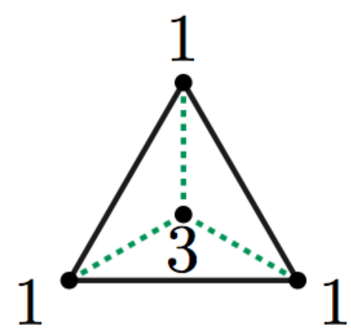
- Max leaf height = height of tree.

Visualizing and Bounding Trees

- Easier to view face on:



- Face tree face on:

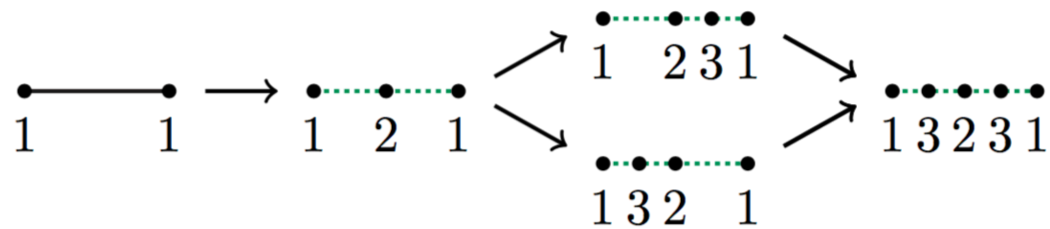


- Bound: $h \leq 6$ sufficient to avoid (4,6) divisors.

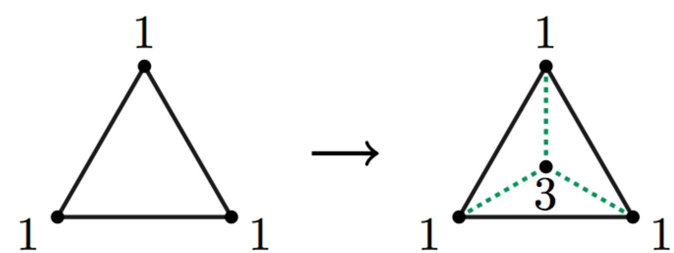
Sung's Talk.

Classifying Bounded Trees

- All 5 $h \leq 3$ edge trees.



- Both $h \leq 3$ face trees.



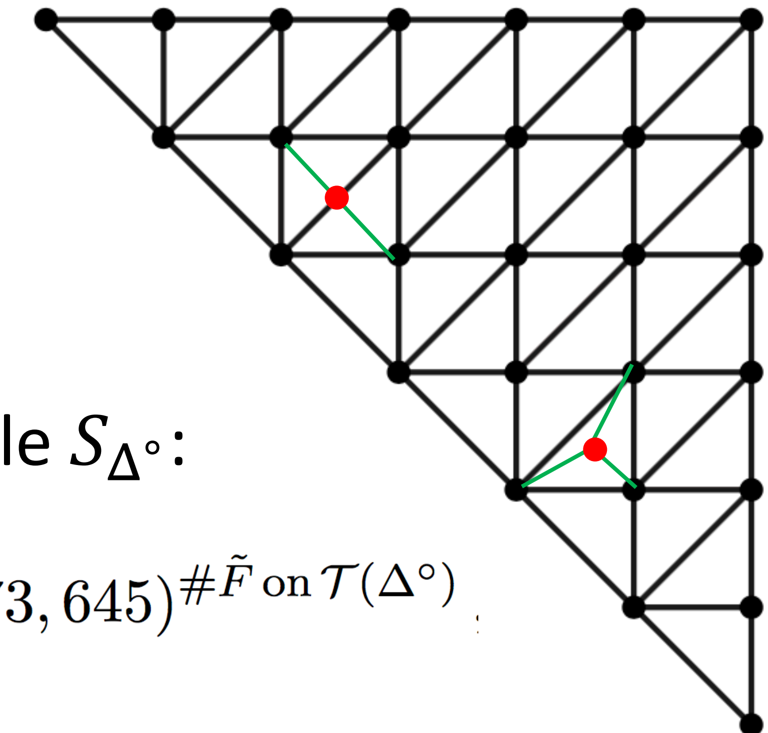
- # for $h \leq N$:

N	# Edge Trees	# Face Trees
3	5	2
4	10	17
5	50	4231
6	82	41,873,645

Forests and Landscapes

- Construction: Make Trees Into Forests

  0) Pick FRST $T(\Delta^\circ)$
  1) Face tree on each face.
  2) Edge tree on each edge.



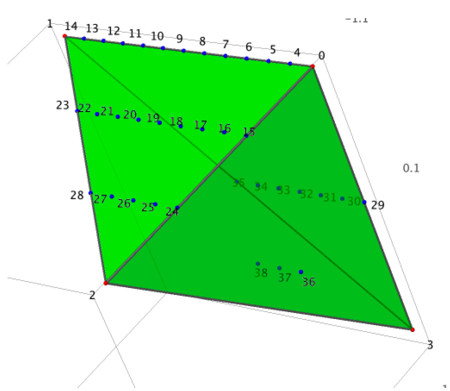
- # geom. in resulting ensemble S_{Δ°:

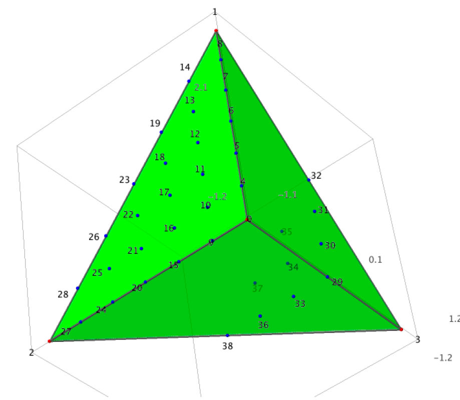
$$|S_{\Delta^\circ}| = 82^{\#\tilde{E}\,\mathrm{on}\,\mathcal{T}(\Delta^\circ)} \times (41,873,645)^{\#\tilde{F}\,\mathrm{on}\,\mathcal{T}(\Delta^\circ)}$$

- Shown facet: $\#E = 63,\ \#F = 36.$

The Big Ones

- Two polytopes Δ_1° and Δ_2° give the biggest ensembles.





- Very large numbers of geometries.

Sung's Talk.

$$|S_{\Delta_1^\circ}| = \frac{2.96}{3} \times 10^{755} \qquad |S_{\Delta_2^\circ}| = 2.96 \times 10^{755}$$

- All others have $|S_{\Delta^\circ}| \leq 3.28 \times 10^{692}$.

Universality

[J.H., Long, Sung]

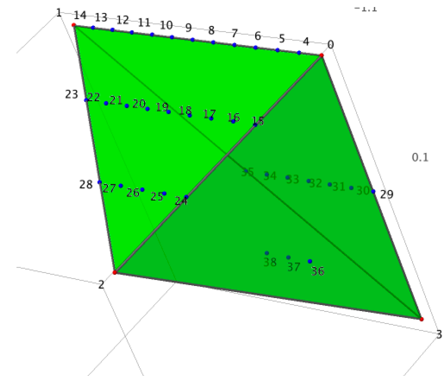
Approaching Universality

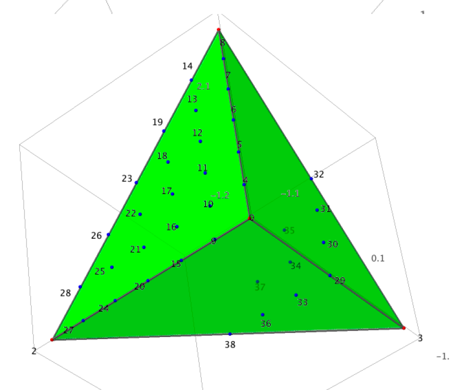
- Assumption A_i. Property P_i.

- If $A_i \to P_\mathrm{i}$,   then $P(P_i) \geq P(A_i)$.

- Goal: find pairs (A_i, P_i) that

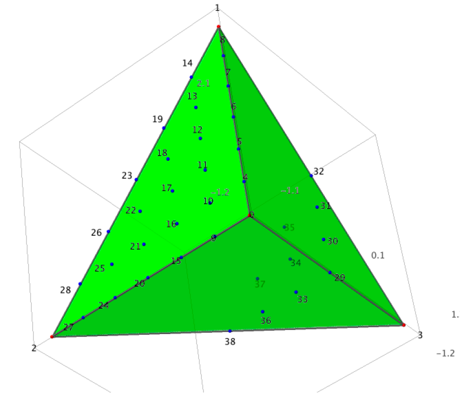
    1) maximize $P(A_i)$
    2) maximize   interestingness(P_i).





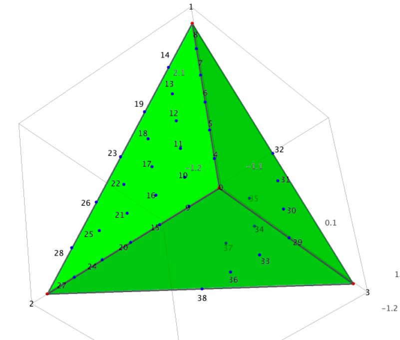
- Here: A_i tree geometry, P_i physical.
  Geometry → Physics.

NHC Universality



- Consider Δ° with interiors on facet F.
- A = $h \geq 2$ leaf on F.   P = NH7 on all F interiors.
- Simple proof (w/ poly normality):   A→P
- Probability: $$P(\mathrm{NHC\ in\ } S_{\Delta^\circ}) \geq 1 - \frac{1}{|S_{\Delta^\circ}|}$$
- Probability in the big ones: $$P(\mathrm{NHC\ in\ } S_{\Delta_1^\circ}) \geq 1 - 1.01 \times 10^{-755}$$ $$P(\mathrm{NHC\ in\ } S_{\Delta_2^\circ}) \geq 1 - .338 \times 10^{-755}$$

Minimal Gauge Universality



- ***Theorem:*** A leaf built on E_8 roots with height $h = 1,2,3,4,5,6$ has Kodaira fiber $F = II^*, IV^*_{ns}, I^*_{0,ns}, IV_{ns}, II, -$ and geometric gauge group $G = E_8, F_4, G_2, SU(2), -, -$, respectively.

- $A_3 \rightarrow P_3$**,** let H_i be # leaves of height i above E_8 roots.

  A_3: every vertex-containing simplex has a face tree and there is a height $h \geq 5$ face tree somewhere on big facet F.

  P_3: $G \geq E_8^{10} \times F_4^{18} \times U^9 \times F_4^{H_2} \times G_2^{H_3} \times A_1^{H_4}$ $\quad U \in \{G_2, F_4, E_6\}$

  $rk(G) \geq 160 + 4H_2 + 2H_3 + H_4$

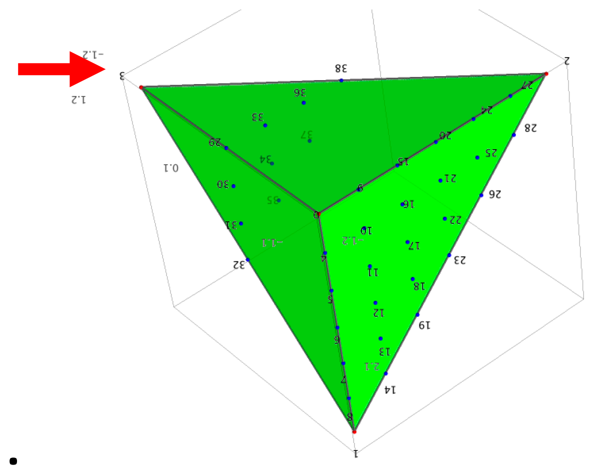
- Probability: $P(P_3) \geq P(A_3) \geq .999995$

Random Sampling Results

- Random face and edge trees on pushing triangulation.

- Millions of random samples:

  1) ≥ 36 of 38 leaves on ground have E8
  2) leaves on ground only $\in \{E6, E7, E8\}$.
  3) E6 only on one special point:
     with prob $\sim 1/2000$,
     $g_4 = m^2$,
     with $m = (-2,0,0)$.



- When E6? Supervised machine learning.

Nelson's Talk. **(see also, Ruehle's talk for ML!)**

Conclusions

- $\frac{4}{3} \times 2.96 \times 10^{755}$ F-theory geometries.
  Connected CY moduli space.

$$P(\text{NHC in } S_{\Delta_1^\circ}) \geq 1 - 1.01 \times 10^{-755}$$

- NHC Generic: $P(\text{NHC in } S_{\Delta_2^\circ}) \geq 1 - .338 \times 10^{-755}$
- Large Geometric Gauge Generic:

$$G \geq E_8^{10} \times F_4^{18} \times U^9 \times F_4^{H_2} \times G_2^{H_3} \times A_1^{H_4} \qquad rk(G) \geq 160 + 4H_2 + 2H_3 + H_4$$

- Q: how do you do anything with 10^{755} of something?

  Algorithmic universality.
  Derived from construction algorithm,
  not constructed ensemble.

Thanks!

296231521306658847673529878342496866080378021782076722241462425150065298508845621155695644076004524635389132
997146097067111728163388031396056782083347614866566976655879021984012793769157213779169816753325335978325856
186523598065404800375507248464342388511047235006655822119955724586948796896518847577026036417026840946433790
956016724443830558425085959438371548897609540032670520282917998350399793115958231423494399655849101565013413
100994870230661418306662241733727360211923208425845972788270516755913617371792570199269731190754783864008989
335957900232620795965279395190240666076073359738714472109475716934497800719015826299673405211214139176513510
727323709162299991669746741171716096000000000000000000000000000000000000000000000000000000000000000000000000

Caveats and Technicalities

- Flux: Bousso-Polchinski story on top of this.

  h^{11} large $\rightarrow \chi$ likely large. Should be fine.

- Morphisms: $\exists Z_3$, but that seems to be it.

- Moduli: (4,6) divisor pathology from Hayakawa-Wang

  Always avoid, by construction.

- Enlarging the set: mix face, edge blow-ups, e.g.

  Weaken height bound.

  Compare to new Monte Carlo of **[Taylor, Wang]?**