Calabi-Yau Fourfolds with non-trivial Three-Form Cohomology

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String Pheno 2017

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Motivation

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F-theory effective action on elliptically fibered Calabi-Yau fourfold Y_4:

$\mathcal{N} = 1$ gauged supergravity in $(3+1)$ dimensions

$$S^{(4)} = \int_{\mathcal{M}_{3,1}} \frac{1}{2} R^{(4)} * 1 - K^F_{I\bar{J}} \mathcal{D}M^I \wedge * \mathcal{D}\bar{M}^J$$
$$- \frac{1}{2} Re(f)_{\Lambda\Sigma} F^\Lambda \wedge * F^\Sigma - \frac{1}{2} Im(f)_{\Lambda\Sigma} F^\Lambda \wedge F^\Sigma$$

Massless spectrum includes complex scalars from non-trivial harmonic three-forms of Y_4

$\Rightarrow$ What can we learn about their dynamics?

$\Rightarrow$ Can we construct explicit examples?

Outline

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- Three-Forms on Calabi-Yau Fourfolds
- Calabi-Yau Fourfolds as Toric Hypersurfaces
- Metric for Three-Form Moduli of a Toric Hypersurface
- Outlook

Geometry and Topology of CY_4

Geometry and Topology of CY_4

Characteristic feature:
$SU(4)$-holonomy

Covariantly constant Weyl-spinor:

$$\nabla\eta = 0, \quad \gamma_9\eta = \eta$$

$\Rightarrow \mathcal{N} = 2$ 3d M-theory vacua!
(no flux)

$\Rightarrow \mathcal{N} = 1$ 4d F-theory vacua!
(no flux)

Hodge diamond:

$$\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 0 & & 0 & & & \\
 & & 0 & & h^{1,1} & & 0 & & \\
 & 0 & & h^{2,1} & & h^{2,1} & & 0 & \\
1 & & h^{3,1} & & h^{2,2} & & h^{3,1} & & 1 \\
 & 0 & & h^{2,1} & & h^{2,1} & & 0 & \\
 & & 0 & & h^{1,1} & & 0 & & \\
 & & & 0 & & 0 & & & \\
 & & & & 1 & & & &
\end{array}$$

Expansion of the Three-Form

Consider scalar three-form moduli arising from $C_3 \in H^{2,1}(Y_4) \oplus H^{1,2}(Y_4)$

Kinetic term: $dC_3 \wedge * dC_3$ $\Rightarrow$ Hodge-star depends on moduli!

precisely: $$*\psi = iJ \wedge \psi, \quad \psi \in H^{2,1}(Y_4)\,, \quad J \in H^{1,1}(Y_4)$$

$\Rightarrow$ Choose three-form basis Ψ^l depending on complex structure moduli!

$$\Psi^l(z,\bar{z}) = \tfrac{1}{2} Re(f)^{lm}(\alpha_m - i\bar{f}_{mk}\beta^k) \in H^{1,2}(Y_4)$$

Three-form deformations:

$$C_3 = \sum_{l=1}^{h^{2,1}} N_l \Psi^l + c.c.$$

$\Rightarrow$ $h^{2,1}$ complex scalars N_l

$\Rightarrow$ bosonic part of chiral multiplets

Three-Form Ansatz

Suggested by [Grimm '10]:
$(l, k, m = 1, \ldots, h^{2,1})$

$$\Psi^l = \frac{1}{2} Re(f)^{lm} (\alpha_m - i\bar{f}_{mk}\beta^k) \in H^{1,2}(Y_4)$$

Properties:

- α_l, β^k basis of $H^3(Y_4, \mathbb{R}) \Rightarrow$ topological
- $f_{lm}(z)$ holomorphic (three-form periods)
- Assume: $\beta_l \wedge \beta_m = 0$

Result:

$$\frac{\partial \Psi^l}{\partial z^K} = \frac{\partial \overline{\Psi}^l}{\partial z^K} \sim \Psi^m$$

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Advantage: $\quad {M_{Al}}^k = \int \omega_A \wedge \alpha_l \wedge \beta^k$ topological intersection numbers

$$\Rightarrow \quad \int_{Y_4} \Psi^l \wedge * \bar{\Psi}^k = -\frac{1}{2} Re(f)^{lm} {M_{Am}}^k v^A$$

Goal of our work:
Calculate f and M!

$K^M_{new} \sim Re(N)_l Re(f)^{lm} {M_{Am}}^k v^A Re(N)_k \quad \Rightarrow Im(N)_l$ axionic!

Toric Calabi-Yau Fourfold Hypersurfaces

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Now Calabi-Yau fourfold Y_4 smooth toric hypersurface

Toric divisors D_i, codimension one submanifolds of Y_4 invariant under torus action

Non-trivial three- and two-forms from divisors D_i via Gysin-morphism of their inclusion

$$\iota_i \; : \quad D_i \to Y_4$$ [Danilov;Mavlyutov]

Gysin-Morphism

$$\iota_{i*}: \; H^n(D_i,\mathbb{C}) \xrightarrow{PD} H_{6-n}(D_i,\mathbb{C}) \xrightarrow{(\iota_i{}^*)^{-1}} H_{6-n}(Y_4,\mathbb{C}) \xrightarrow{PD} H^{n+2}(Y_4,\mathbb{C})$$

$\Rightarrow$ Respect Hodge decomposition:

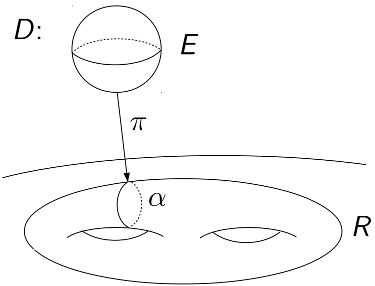
$$\iota_{i*}: H^{0,0}(D_i) \longrightarrow H^{1,1}(Y_4)\,,$$
$$\iota_{i*}: H^{1,0}(D_i) \longrightarrow H^{2,1}(Y_4)\,.$$

For smooth toric Calabi-Yau hypersurfaces Y_4

$$\bigoplus_i H^{0,0}(D_i) \simeq H^{1,1}(Y_4)\,,$$
$$\bigoplus_i H^{1,0}(D_i) \simeq H^{2,1}(Y_4)\,, \quad \bigoplus_i H^{0,1}(D_i) \simeq H^{1,2}(Y_4)\,.$$

$\Rightarrow$ Construct $H^{1,0}(D_i)$ for toric divisors D_i of Y_4!

Divisors with non-trivial One-Forms



$$H^{1,0}(D) \simeq H^{1,0}(R)$$

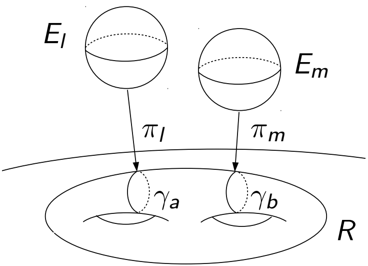
For $H^{1,0}(D) \neq 0$:

Toric divisor D fibration

Fiber: E toric surface,
($h^{i,j}(E) = 0$ for $i \neq j$)

Base: R Riemann surface

Three-forms of Calabi-Yau Fourfold Hypersurface



Construct three-forms:

$$\psi_{\mathcal{A}} = \iota_{I*}(\gamma_a) \in H^{2,1}(Y_4)$$

with $\mathcal{A} = (I, a)$

$\iota_I : \quad D_I \to Y_4 \,, \quad \gamma_a \in H^{1,0}(R)$

$\Rightarrow$ Complex structure dependence of $\psi_{\mathcal{A}}$ determined by $H^{1,0}(R)$

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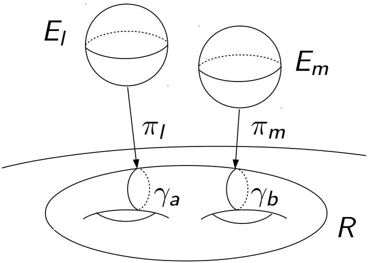

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Normalized basis: $\gamma_a = \alpha_a + i\, f_{ab}(z)\beta^b \in H^{1,0}(R)\,, \quad \alpha_a, \beta^b \in H^1(R, \mathbb{Z})$

Metric on $H^{1,0}(R)$: $-i \int_R \gamma_a \wedge \bar{\gamma}_b = 2 \cdot Re(f)_{ab} > 0\,, \quad f_{ab} = f_{ba}$

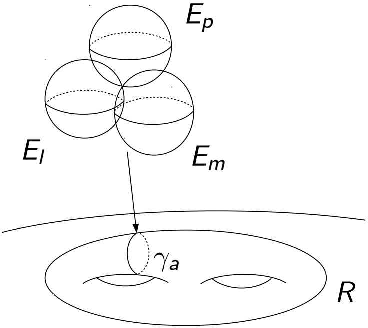
Three-Form Metric for Fourfold Hypersurface

Recall metric on $H^{2,1}(Y_4)$

$$Q_{\mathcal{A}\mathcal{B}} = \int_{Y_4} \psi_{\mathcal{A}} \wedge *\bar{\psi}_{\mathcal{B}} = -i \int_{Y_4} J \wedge \psi_{\mathcal{A}} \wedge \bar{\psi}_{\mathcal{B}}\,, \quad J \in H^{1,1}(Y_4)$$

Two- and three-forms from toric divisors via Gysin-map

$\Rightarrow$ Triple intersection of toric divisors!



Only divisors that fiber over same R contribute!

$$D_l \cap D_m \cap D_p = M_{lmp} \cdot R$$

with $M_{lmp} = \# E_l \cap E_m \cap E_p$
(Generalized sphere tree)

Three-Form Metric for Fourfold Hypersurface

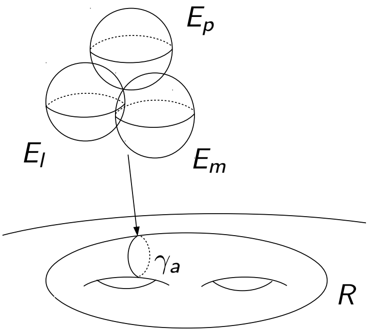
Metric on $H^{2,1}(Y_4)$

$$Q_{\mathcal{A}\mathcal{B}} = 2v^I\, M_{Imp}\, Re(f)_{ab}$$

$\mathcal{A} = (m, a)$, $\mathcal{B} = (p, b)$

Reorder indices to find previous result

$$Q_{\mathcal{A}\mathcal{B}} = 2v^{\Sigma} M_{\Sigma\mathcal{A}}{}^{\mathcal{C}}\, Re(f)_{\mathcal{C}\mathcal{B}}$$



$v^{\Sigma} M_{\Sigma\mathcal{A}}{}^{\mathcal{C}}$ linear dependence on Kähler moduli v^{Σ}

$Re(f)_{\mathcal{C}\mathcal{B}}$ complicated dependence on complex structure moduli z^K

$\Rightarrow$ Toric data exchanged by mirror symmetry!

Outlook

Next step:

Discuss F-theory physics on elliptically fibered CY_4 and their weak coupling limit on CY_3

Construct examples with three-forms such that weakly coupled IIB has

- Wilson line moduli on a $D7$ wrapping a four-cycle of CY_3 (axions)
- Odd moduli, scalars from B_2, C_2 (axions)

$\Rightarrow$ Metric on $H^{2,1}(Y_4)$ determines decay-constants!

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Future directions:

- Construct bases with three-forms (bulk $U(1)$)
- Generalize to complete intersections
- Include G_4-flux and calculate superpotential
- ...

Thank you for your attention! Questions?