Heterotic Flux Compactifications

Mario Garcia-Fernandez

Instituto de Ciencias Matemáticas, Madrid

String Pheno 2017
Virginia Tech, 7 July 2017

Based on arXiv:1611.08926, and joint work with Rubio, Tipler, Shahbazi (Math. Annalen & ongoing).

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... Again???

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Outline

- This talk is about the internal geometry of four-dimensional $N = 1$ compactifications of heterotic string theory with flux H.

Hull '86, Strominger '86

- For Calabi-Yau compactifications ($H = 0$, ϕ constant), topology of internal space determines the field content of the $4D$-supergravity.

$$h^{1,1} \sim \text{sizes of } 2\text{-cycles}, \; h^{2,1} \sim \text{sizes of } 3\text{-cycles}.$$

Candelas-Horowitz-Strominger-Witten '85

- Furthermore, Yau's Theorem '76 allowed the application of powerful methods from algebraic and Kähler geometry leading to important advances.

- However, the many massless scalar fields arising from generic CY compactifcations called for stabilising mechanisms.

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- In the presence of fluxes, the geometry is not Kähler and classical methods no longer apply. Furthermore, $H \neq 0$ requires holomorphic bundles which make the analysis really complicated

$$dH = \alpha'(\mathrm{tr}\, R \wedge R - \mathrm{tr}\, F \wedge F).$$

- Recent developments in the study of
  - geometry of equations of motion (GF '13, Coimbra-Minasian-Triendl-Waldran '14)
  - heterotic T-duality (Baraglia-Hekmati '13, GF '16)
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Moduli

In 1986 Hull and Strominger characterized warped 4d compactifications of the heterotic string, with $N = 1$ supersymmetry, non-zero flux $H \neq 0$, and non-constant dilaton ϕ.

Given by an $SU(3)$-structure (ψ, ω) with almost complex structure $J\colon TM^6 \to TM^6$ and metric g, and a gauge field A with field strength F, such that

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\begin{aligned}
d\Omega &= 0,\\
g^{i\bar{j}} F_{i\bar{j}} = 0, \qquad F_{\bar{i}\bar{j}} &= 0,\\
d^*\omega - i(\bar{\partial} - \partial)\log\|\Omega\| &= 0,\\
2i\partial\bar{\partial}\omega - \alpha'(\operatorname{tr} R \wedge R - \operatorname{tr} F \wedge F) &= 0,
\end{aligned}
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where $\Omega = e^{\phi}\psi$, $H = i(\bar{\partial} - \partial)\omega$, $\phi = \log\|\Omega\|$

In this talk, first order equations in α'-expansion taken as exact (my apologies). Imposing the equations of motion, is equivalent to

$$g^{i\bar{j}} R_{i\bar{j}} = 0, \; R_{\bar{i}\bar{j}} = 0.$$

Reminiscent of $R = R_{\nabla^-}$ in α'-expansion.

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Theorem (_____-Rubio-Tipler '15)

The Hull-Strominger system (below) is elliptic. Therefore, the moduli space is finite-dimensional.

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Remark: does not take into account b-fields. Where are they?

Canonically attached to M and the gauge bundle V, there is a set of *string classes* $H^3_{str}(V,\mathbb{R})$ (Redden '11). Fixing (H_0, ∇_0, A_0) a solution of the Bianchi identity, yields identification $H^3_{str}(V,\mathbb{R}) \cong H^3(M,\mathbb{R})$

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dH_0 = \alpha' \operatorname{tr} R_{\nabla_0}\wedge R_{\nabla_0} - \alpha' \operatorname{tr} F_{A_0}\wedge F_{A_0}.
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Flux map

A choice of *integral string class* $\sigma \in H^3_{str}(V, \mathbb{Z})$, corresponds to a string structure (up to homotopy) Killingback '87, Witten '87 $\sim$ choice of quantization scheme for heterotic Σ-model Freed '86, Waldorf '13.

There is a well-defined map *flux* on moduli, so that (a first approximation to) moduli of heterotic compactifications is given by $M^{\sigma}_{HS} = flux^{-1}(\sigma)$

$$\mathcal{M}_{HS} \xrightarrow{flux} H^3_{str}(V, \mathbb{R}).$$

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M^{σ}_{HS} corresponds to a moduli space of natural Killing spinor equations in generalized geometry, on a Courant algebroid E_σ determined by $\sigma \in H^3_{str}(V, \mathbb{Z})$.

Idea: infinitesimally

$$\delta flux(\dot{\Omega}, \dot{\omega}, \dot{\nabla}, \dot{A}) = [\delta(d^c_J \omega) - 2\alpha' \operatorname{tr} \dot{\nabla} \wedge R_\nabla + 2\alpha' \operatorname{tr} \dot{A} \wedge F_A] \in H^3(M, \mathbb{R}).$$

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Now, $\delta flux = 0$, implies there exists a 2-form b on M such that

$$db = \delta(d^c_J \omega) - 2\alpha' \operatorname{tr} \dot{\nabla} \wedge R_\nabla + 2\alpha' \operatorname{tr} \dot{A} \wedge F_A.$$

- (ω, b, ∇, A) determines generalized metric on

$$E_\sigma = T \oplus T^* \oplus \operatorname{End} T \oplus \operatorname{End} V$$

- Having added parameters $b \in \Omega^2(M)$, need to add symmetries: B-field symmetries.
- Imposing equations are preserved by B-field symmetries: automorphism group of E_σ.

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Kähler moduli

Fixing the complex structure X on M, and consider the Aeppli cohomology group

$$H_A^{1,1}(X) = \frac{\text{Ker } i\partial\bar{\partial}\colon \Omega^{1,1} \to \Omega^{2,2}}{\text{Im } \partial \oplus \bar{\partial}\colon \Omega^{0,1} \oplus \Omega^{1,0} \to \Omega^{1,1}}.$$

Fixing the holomorphic bundle structure on V and TX, the equation

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implies that there is a well-defined map

$$[(\dot{\Omega}, \dot{\omega}, \dot{\nabla}, \dot{A})] \to [\dot{\omega} + ib - 2\alpha' \operatorname{tr} s_1 \wedge R_\nabla + 2\alpha' \operatorname{tr} s_2 \wedge F_A] \in H_A^{1,1}(X)$$

for suitable infinitesimal complex gauge transformations s_1, s_2.

Kähler moduli of a heterotic flux compactification is $H_A^{1,1}(X)$
($\sim$ d-closed $(1,1)$-currents $\supset$ analytic 2-cycles)

Remark: assuming $\partial\bar{\partial}$-Lemma holds, De la Ossa-Svanes '14 and Anderson-Gray-Sharpe '14 propose $H_{\bar{\partial}}^{1,1}(X)$ as Kähler moduli.

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for suitable infinitesimal complex gauge transformations s_1, s_2.

Kähler moduli of a heterotic flux compactification is $H_A^{1,1}(X)$
($\sim$ d-closed $(1,1)$-currents $\supset$ analytic 2-cycles)

Remark: assuming $\partial\overline{\partial}$-Lemma holds, De la Ossa-Svanes '14 and Anderson-Gray-Sharpe '14 propose $H_{\overline{\partial}}^{1,1}(X)$ as Kähler moduli.

Twisted Heterotic Compactifications

A key ingredient for the description of Hull-Strominger system using generalized geometry in arXiv:1611.08926 is the treatment of the dilaton field ϕ using *Dirac generating operators* (Ševera, Alekseev-Xu, unpublished '01). In this approach, ϕ may be only locally defined, with field-strength given by a closed 1-form in the internal manifold M.

'Assuming $H^1(M)$ is trivial, this implies ... a globally defined scalar field ϕ. If $H^1(M)$ is not trivial, there are interesting new possibilities [30].'

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Common sector: A twisted heterotic compactification to $10 - 2n$ dimensions is given by a $SU(n)$-structure (ω, Ψ) on M^{2n} satisfying

$$\begin{aligned} d\Psi - \theta \wedge \Psi &= 0, \\ d\theta &= 0, \\ dd^c\omega &= 0. \end{aligned} \tag{1}$$

where $\theta = Jd^*\omega$ is the Lee form of the Hermitian structure.

- Mild class of non-geometric backgrounds, which violate Maldacena-Nuñez Theorem.
- Very small moduli space.

Remark: If M is compact and θ exact, then $H = 0$, $\theta = 0$ and g is a Calabi-Yau metric.

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Example: Twisted compactification to $6d$ on $M^4 = S^3 \times S^1$.

Identify $M \cong SU(2) \times U(1)$, with Lie algebra

$$\mathfrak{su}(2) \oplus \mathbb{R} = \langle e_1, e_2, e_3, e_4 \rangle,$$

where

$$de^1 = e^2 \wedge e^3, \quad de^2 = e^3 \wedge e^1, \quad de^3 = e^1 \wedge e^2, \quad de^4 = 0.$$

Using T-duality with parameters, obtain all homogeneous solutions on M, given by $(\beta, \gamma, \ell, \tau) \in \mathbb{R}_{>0} \times \mathbb{R}^3$.

$$g = e^{1'} \otimes e^{1'} + e^2 \otimes e^2 + e^3 \otimes e^3 + \frac{\gamma^2}{\beta^2} e^4 \otimes e^4$$

$$H = -\beta e^{1'} \wedge de^{1'}, \qquad \Psi = (e^{1'} + i\frac{\gamma}{\beta} e^4) \wedge (e^2 + ie^3)$$

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where $e^{1'} = e^1 + \frac{\ell}{\beta} e^4$.

Remark: These are homogeneous primary Hopf surfaces. Moduli of homogeneous complex structures $\mathcal{M}_{cx} = \mathbb{C}$ (Hasegawa-Kamishima).

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Moduli $\mathcal{M}$ (twisted common sector)

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Remarks:

- $\partial\overline{\partial}$-Lemma does not hold (in fact, $H^{1,1}_{\overline{\partial}}(X)=0$). Kähler moduli given by non-topological, transcendental quantities
- Standard compactifications on $K3$ have (real) 82-dimensional moduli.
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Heterotic T-duality

Fix a heterotic flux compactification $(\Omega, \omega, \nabla, A)$ with gauge group G, let $P = P_g \times_M P_V$ (a $Spin(6, \mathbb{R}) \times G$-bundle).

Definition: The string class of the heterotic flux compactification $(\Omega, \omega, \nabla, A)$ is

$$[\hat{H}] = [p^* d^c \omega - \alpha' CS(\nabla) + \alpha' CS(A)] \in H^3(P, \mathbb{R}),$$

where

$$CS(A) = tr(A \wedge F_A + \frac{1}{6} A \wedge [A \wedge A])$$

Assuming that M and V have torus action T^k, can consider T^k-invariant string classes:

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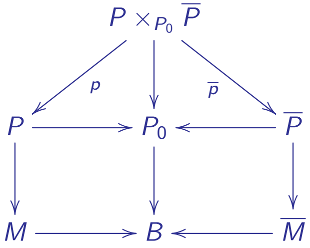
Let (P, σ), $(\overline{P}, \overline{\sigma})$ a pair as before, with commuting diagram ($B = M/T^k = \overline{M}/\overline{T}^k$ and $P_0 = P/T^k = \overline{P}/\overline{T}^k$)



Definition (Baraglia-Hekmati '15): (P, σ) is T-dual to $(\overline{P}, \overline{\sigma})$ if exists an invariant 2-form F on $P \times_{P_0} \overline{P}$ which is 'non-degenerate on the fibres' of $P \times_{P_0} \overline{P} \to P_0$ and representants $\hat{H} \in \sigma$ and $\hat{\overline{H}} \in \overline{\sigma}$, such that $p^* \hat{H} - \overline{p}^* \hat{\overline{H}} = dF$.

Theorem (GF '16)

Solutions of the Hull-Strominger system are preserved by Heterotic T-duality.

Remarks: Existence of T-dual requires flux quantization $\sigma \in H^3_{str}(V, \mathbb{Z})$. Previous partial check by (Evslin-Minasian '08).

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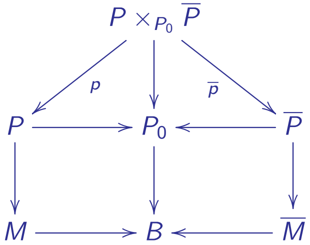

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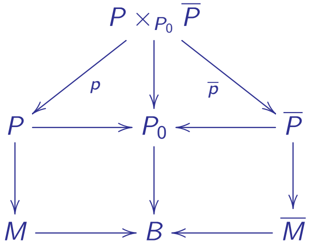

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Thank you!