Heterotic Line Bundle Models on Elliptically Fibered Calabi-Yau Threefolds and the F-theory Duals

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Based on

A. Braun, CB, A. Lukas arXiv:1706.07688

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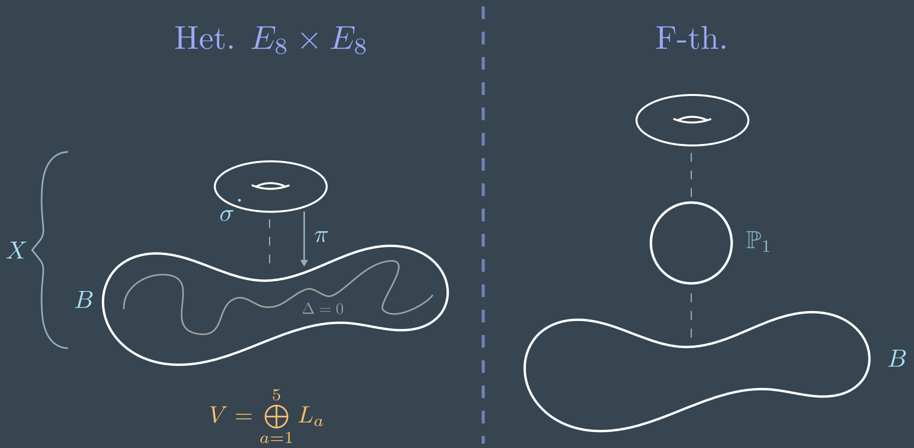
Outline

- Motivation
- Heterotic line bundle models on elliptic threefolds.
  - Interesting aspects of building these models.
  - Resulting phenomenologically interesting models. Generic features.
- Generalities on F-theory duals.
  - General observations.
  - Some concrete aspects of the duality.
- Summary and outlook.

Motivation

- What are generic features of line bundle models on elliptic threefolds? How do they compare to models on other common spaces (e.g. simple CICYs)?
- Line bundle models are important part of moduli space. We should know about their dual pictures.
- Dual picture may be useful for computations. Maybe five-branes, Yukawa couplings?

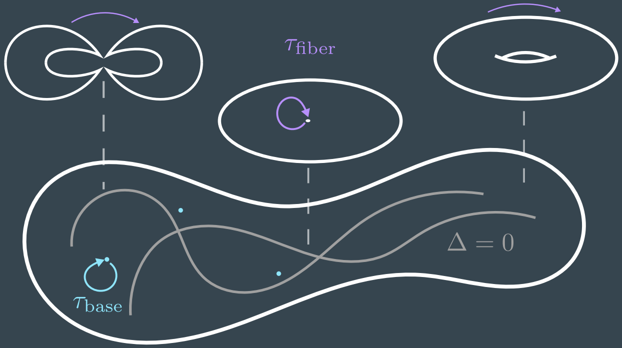
Setup


Het. $E_8 \times E_8$
F-th.
X
B
σ
π
$\Delta = 0$
$V = \bigoplus_{a=1}^{5} L_a$
$\mathbb{P}_1$
B

Aspect 1: Wilson line and involutions

Key points:

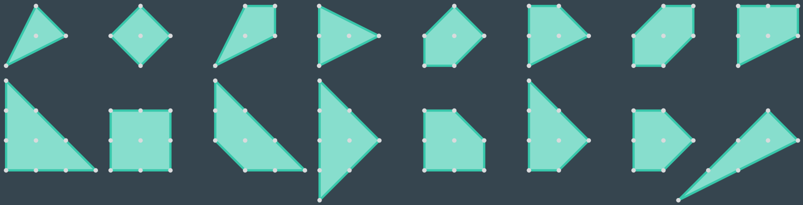
- Quotient by involution $\tau_X^{(\text{free})}$ to introduce Wilson line to break $\mathrm{SU}(5)$. Simple choice is $\tau_X = \tau_{\text{fiber}} \circ \tau_{\text{base}}$.
- Tune second section $\Rightarrow$ resolve.
- τ_{base} only fixed points $\Rightarrow$ can arrange τ_X to be free.



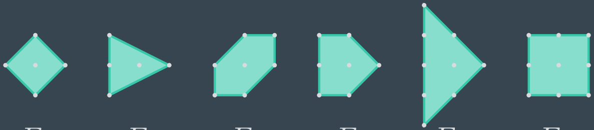
Donagi et al 99.

Aspect 2: Base classification

toric $\supset$ weak Fano: 16 reflexive polytopes



$\supset \tau_{\text{base}}$ only fixed points: 6 polytopes



F_2 ($\mathbb{P}^1 \times \mathbb{P}^1$) F_4 ($\mathbb{F}_2$) F_7 (dP_3) F_9 F_{13} F_{15}

Morrison and Taylor 12; Halverson and Taylor 15.

Aspect 3: Cohomology computations

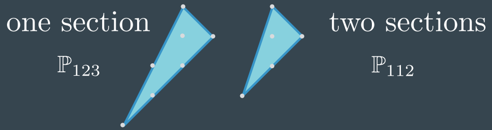
Two possible ways to compute line bundle cohomologies.

1. Leray spectral sequence relates cohomologies to base space cohomologies of higher direct images.

$$\pi_* \mathcal{O}(n\sigma) = \mathcal{O}_B \oplus K_B \oplus \left(K_B^{\otimes 2} \oplus K_B^{\otimes 3} \oplus \ldots \oplus K_B^{\otimes n} \right)^{\oplus 2}$$

2. In two section case, can realise as toric hypersurface with different fiber ambient space $\mathbb{P}_{112}$.

   Then cohomologies from that of toric ambient space (using **cohomCalg**) and Koszul spectral sequence.



Donagi et al 04; Andreas and Curio 07; Blumenhagen et al 10; Morrison and Park 12; Grimm and Weigand 10; Mayrhofer et al 13.

Scan results

base	F_2	F_4	F_7	F_9	F_{13}	$F_{15}^{(a)}$	$F_{15}^{(b)}$
no. models	0	0	54	24	≥ 46	≥ 236	≥ 84

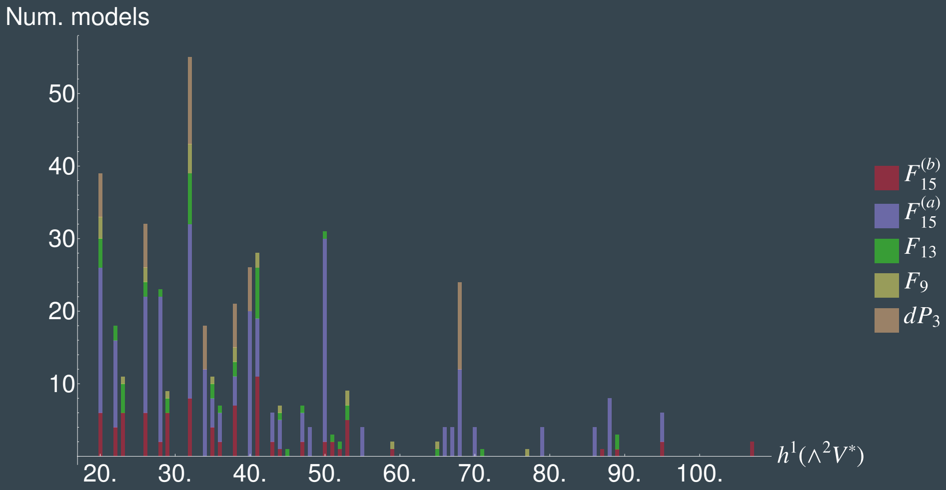
Table: $\mathrm{SU}(5)$ GUT models, 3 generations, invariant under involution.

Generic features of models:

- ≥ 1 $\mathbf{10}$-$\overline{\mathbf{10}}$ pairs and ≥ 20 $\mathbf{5}$-$\overline{\mathbf{5}}$ pairs.
- ~ 100 bundle moduli (singlets).

Large numbers of vector-like pairs related to complicated ambient space cohomology, compared with projective spaces.

Scan results



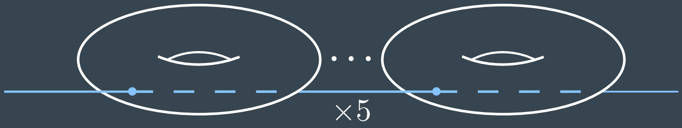
Generalities on F-theory duals of line bundle models

Observations:

- For line bundle sums: flatness on the fiber $\Rightarrow$ zero chirality.

$$\text{ind}\left(\bigoplus_a \mathcal{O}(\vec{k}_a)\right) = \sum_a d_{IJK} k_a^I k_a^J k_a^K = 0 \;\text{ as }\; d_{ijk} = 0\,.$$

- Flatness on the fiber $\Rightarrow$ spectral cover sheets are trivial.
- Spectral cover description not the natural one.



So:

- General line bundle sums seem to require non-flat duality.
- We first conjecture duals of line bundle sums flat on fiber.

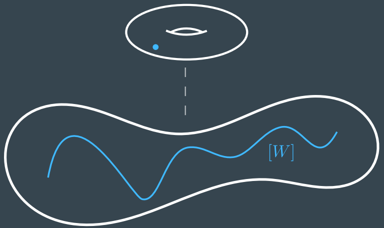
Friedman et al 97.

Generalities on concrete F-theory duals

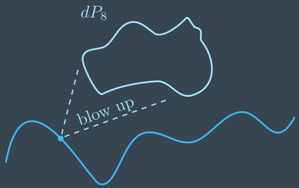
- Bundles flat on fibers ⇒ require five-branes wrapping curves in base to fill out anomaly condition.

$$c_2(X) - c_2(V) = [W] \supset -12K_B \, .$$

- Five-branes in base go to blow-ups in F-theory ⇒ need to blow-up F-theory fourfold.



Het.



F-th.

- On F-theory side, have two E_8 stacks - broken only by flux.

Morrison and Vafa 96; Rajesh 99; Andreas and Curio 99.

Summary and outlook

- Scanned over large class of heterotic line bundle models on elliptic threefolds (SU(5) broken with Wilson line and with three generations).
- Generic features of these models include large numbers of vector like pairs. Seems to be due to relatively complicated ambient space cohomology.
- Currently finalising F-theory dual picture.

Backup slides

Two section model with fiber in $\mathbb{P}_{112}$ given by

$$x_1^2 = x_2^2(x_3^4 + b_2 x_3^2 x_4^2 + b_3 x_3 x_4^3 + b_4 x_4^4) \text{ with } b_3 = 0\,,$$

where weight system is

x_1	x_2	x_3	x_4
1	1	0	0
2	0	1	1

Sections are at

$$x_4 = x_1 \pm x_2 x_3^2 = 0\,,$$

and use $b_3 = 0$ for second section to sit at two-torsion point.
The involution is given by

$$(x_1, x_2, x_3, x_4) \to (-x_1, x_2, x_3, -x_4)\,.$$

Backup slides

multiplet	$S(U(1)^5)$ charge	associated L	contained in
$\mathbf{10}_a$	$\mathbf{e}_a$	L_a	V
$\overline{\mathbf{10}}_a$	$-\mathbf{e}_a$	L_a^*	V^*
$\bar{\mathbf{5}}_{a,b}$	$\mathbf{e}_a + \mathbf{e}_b$	$L_a \otimes L_b$	$\wedge^2 V$
$\mathbf{5}_{a,b}$	$-\mathbf{e}_a - \mathbf{e}_b$	$L_a^* \otimes L_b^*$	$\wedge^2 V^*$
$\mathbf{1}_{a,b}$	$\mathbf{e}_a - \mathbf{e}_b$	$L_a \otimes L_b^*$	$V \otimes V^*$

Table: *A list of $SU(5) \times S(U(1)^5)$ multiplets in the low-energy theory and their associated line bundles. The multiplicity of each multiplet is computed by the cohomology $h^1(X, L)$ of the associated line bundle L.*