Gauge Backgrounds and Zero-Modes in F-Theory

- arXiv:1402.5144 with **M. Bies, C. Mayrhofer, C. Pehle**
- arXiv:1706.04616 with **M. Bies, C. Mayrhofer**
- Anomalies and Algebraic Cycles:
  arXiv:1706.08528 with **M. Bies, C. Mayrhofer**

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What and Why?

Consider a 4d F-theory compactification on a smooth elliptic 4-fold $\hat{Y}_4$.

We will develop a framework to

- parametrise gauge backgrounds *beyond the field strength*
- compute the spectrum of massless states *beyond the chiral index*

This is important

- for phenomenology:

  Need to know the exact massless spectrum for model building

  $\rightsquigarrow$ number of Higgs doublets

  $\rightsquigarrow$ amount of light vectorlike exotics

- conceptually:

  Vectorlike massless spectrum is part of characteristic data of a vacuum

  $\rightsquigarrow$ affects RG flow e.g. of gauge couplings

  $\rightsquigarrow$ enters Higgsings and transitions between different

  $\rightsquigarrow$ depends on moduli and goes beyond rigid data

How?

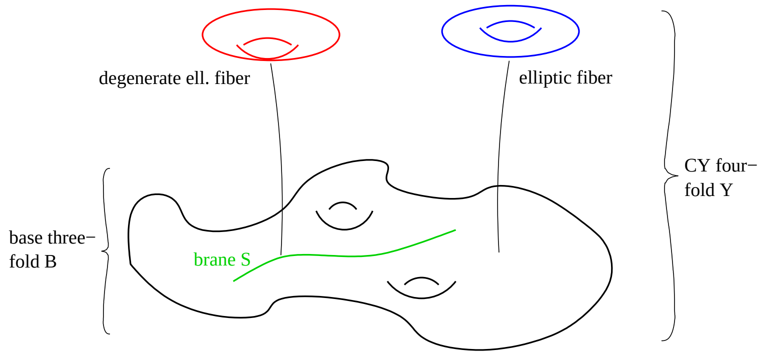
3 steps to compute your spectrum

1. **Parametrise the gauge background**
   $\leftrightarrow$ Chow group of $2_{\mathbb{C}}$-cycles modulo rational equivalence

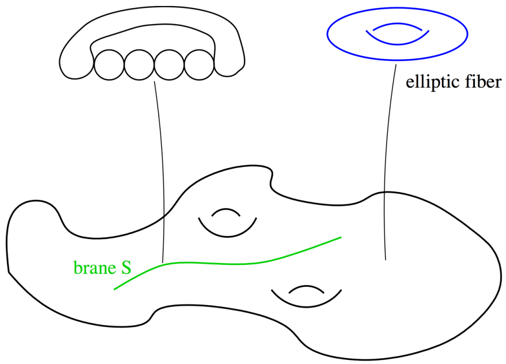
2. **Extract sheaf cohomology groups** counting the massless matter
   $\leftrightarrow$ Intersection theory in the Chow ring

3. **Evaluate the sheaf cohomologies**
   $\leftrightarrow$ f.p. graded S-modules and their Ext groups see talk by Martin Bies

F-theory

Variation of axio-dilaton τ	(singular) torus-fibration Y_4 over B_3 [Vafa'96],...	

Singularity over 7-brane requires resolution $Y_4 \to \hat{Y}_4$

7-brane on $2_{\mathbb{C}}$-cycle S	codimension one singularity algebra $\mathfrak{g}$	

Resolving Y_4 $\leftrightarrow$ Going to Coulomb branch in M-theory

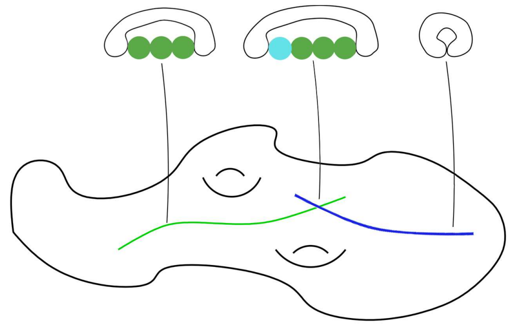
F-theory matter(s)

Localised matter in repr. $\mathbf{R}$ at intersection of 7-branes over curve $C_{\mathbf{R}}$

Origin of matter: [Katz,Vafa'96],[Witten'96]...
wrapped M2-branes on extra $\mathbb{P}^1$s over $C_{\mathbf{R}}$

matter surface $S(\mathbf{R})$

codimension two fibre splitting



Representation $\mathbf{R} \leftrightarrow$ states/weights labeled by $a = 1, \ldots, \dim(\mathbf{R})$

$S^a(\mathbf{R})$: linear combination of $\mathbb{P}^1$s over $C_{\mathbf{R}}$ associated with weight $\beta^a(\mathbf{R})$

Recent systematic studies of matter include [Grassi,Morrison'00 &'11], [Morrison,Taylor'11], [Grassi,Halverson,Shaneson'13],[Cvetič,Klevers,Piragua,Taylor'15], [Anderson,Gray,Raghuram,Taylor'15], [Klevers,Taylor'16], [Klevers,Morrison,Raghuram,Taylor'17]

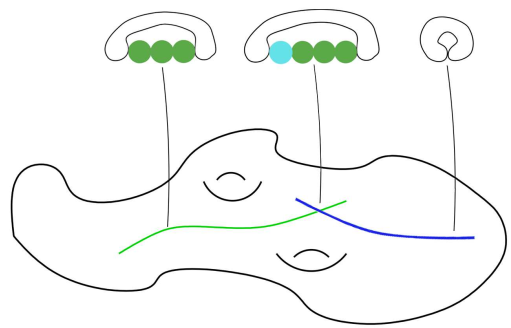
$\rightsquigarrow$ cf talks by Raghuram, Taylor

Step 1:
Describing the gauge background

Fluxes and Chirality

New ingredient in 4d F-theory beyond geometry of fibration:

Non-trivial gauge backgrounds



M2-branes couple to M-theory
3-form gauge potential C_3:

- $C_3 \simeq C_3 + d\Lambda_2$ is higher form gauge potential
- $G_4 = dC_3$ field strength $\longrightarrow$ flux $[G_4] \in H^{2,2}(\hat{Y}_4)$

Chiral index:

$$\nu_+(\beta^a(\mathbf{R})) - \nu_-(\beta^a(\mathbf{R})) = \int_{S^a(\mathbf{R})} [G_4] \qquad S^a(\mathbf{R}) : \text{ matter surface}$$

[Donagi,Wijnholt'09]; [Braun,Collinucci,Valandro'11] [Marsano,S-Nameki'11],

[Krause,Mayrhofer,TW'11], [Grimm,Hayashi'11], [Krause,Mayrhofer,TW'12]

[Cvetič,Grimm,Klevers'12] [Braun,Grimm,Keitel'13], [Cvetič,Grassi,Klevers,Piragua'13],

[Borchmann,Mayrhofer,Palti,TW'13], [Cvetič,Klevers,Mayorga,Oehlmann,Reuter'15],

[Lin,Mayrhofer,Till,TW'15], [Lin,TW'16], ...

Disclaimer

- What specifies the C_3 'gauge data' beyond the field strength/flux?
- What counts the actual number of $\mathcal{N} = 1$ chiral multiplets?

1) Goal: Understand gauge background in globally consistent setting of elliptic fourfold in language of dual M-Theory

- To avoid dealing with singular spaces requires resolving Y_4 to $\hat{Y}_4$

$\Longrightarrow$ Can only detect abelian gauge background at this stage

Possible approaches towards dealing with fluxes in global singular settings might depart from [Anderson,Heckman,Katz'13] [Collinucci,Savelli'14] [Collinucci,Giacomelli,Savelli,Valandro'15] cf. talk by Valandro

2) Equivalent description of gauge backgrounds via formalism of Cheeger-Simons cohomology on smooth backgrounds studied in [Intriligator,Jockers,Mayr,Morrison,Plesser'12]

Gauge background in F/M-theory

- Field strength G_4 $\leftrightarrow$ $H^{2,2}_{\mathbb{Z}/2}(\hat{Y}_4)$
- Wilson line d.o.f. $\oint C_3$ $\leftrightarrow$ Flat gauge backgrounds

Both data summarized in **Deligne cohomology group** $H^4_D(\hat{Y}_4, \mathbb{Z}(2))$

$$0 \longrightarrow \underbrace{J^2(\hat{Y}_4)}_{\oint C_3 \text{ 'Wilson lines'}} \longrightarrow \underbrace{H^4_D(\hat{Y}_4, \mathbb{Z}(2))}_{\textbf{Deligne cohomology}} \xrightarrow[\textbf{onto}]{\hat{c}_2} \underbrace{H^{2,2}_{\mathbb{Z}}(\hat{Y}_4)}_{\text{field strength } G_4} \longrightarrow 0$$

- Deligne cohomology group $H^4_D(\hat{Y}_4, \mathbb{Z}(2))$ $\longleftrightarrow$ equ. cl. of gauge background
- $H^{2,2}_{\mathbb{Z}}(\hat{Y}_4)$ $\longleftrightarrow$ field strength G_4
- Intermediate Jacobian $\longleftrightarrow$ Wilson lines $\oint C_3$

$$\mathcal{J}^2(\hat{Y}_4) \simeq H^3(\hat{Y}_4, \mathbb{C})/(H^{2,1}(\hat{Y}_4) + H^3(\hat{Y}_4, \mathbb{Z}))$$

[Curio,Donagi'98] [Donagi,Wijnholt'12/13][Anderson,Heckman,Katz'13];

[Intriligator,Jockers,Mayr,Morrison,Plesser'12]

Gauge background in F/M-theory

Practical Parametrisation

Deligne $H_D^4(\hat{Y}_4, \mathbb{Z}(2))$ $\longleftrightarrow$ Chow group $\mathrm{CH}^2(\hat{Y}_4)$

Group of algebraic $2_{\mathbb{C}}$-cycles modulo rational equivalence $= \mathrm{CH}^2(X)$

- Rational equivalence:
  $C_1 \cong C_2 \in Z_p(X)$ if $C_1 - C_2$ is zero/pole of a meromorphic function defined on an $(p+1)$-dimensional irreducible subvariety of X
- Chow group $\mathrm{CH}^k(X)$
  $=$ group of rational equivalence classes of codim. k-cycles
- Special case: $\mathrm{CH}^1(X) = \mathrm{Pic}(X) =$ group of line bundles

Explicit construction of gauge fluxes via algebraic cycles in

[Braun,Collinucci,Valandro'11]

Gauge background in F/M-theory

Idea: [Bies,Mayrhofer,Pehle,TW'14]

Concrete representation of gauge data by specification of element

$$A \in \mathrm{CH}^2(\hat{Y}_4) \text{ with } [A] = G_4 \in H^{2,2}(\hat{Y}_4)$$

via 'refined cycle map' $\hat{\gamma}_2 : \mathrm{CH}^2(\hat{Y}_4) \to H^4_D(\hat{Y}_4, \mathbb{Z}(2))$ e.g. [Esnault,Viehweg'88]

$$\begin{array}{ccccccccc}
 & & & & \overbrace{\mathrm{CH}^{\mathbf{2}}(\hat{\mathbf{Y}}_{\mathbf{4}})}^{\textbf{geometry}} & & & & \\
0 & \longrightarrow & \mathrm{CH}^2_{\mathrm{hom}}(\hat{Y}_4) & \longrightarrow & \mathrm{CH}^{\mathbf{2}}(\hat{\mathbf{Y}}_{\mathbf{4}}) & \xrightarrow{\gamma_2} & H^{2,2}_{alg}(\hat{Y}_4) & \longrightarrow & 0 \\
 & & \downarrow{\scriptstyle AJ} & & \downarrow{\scriptstyle \hat{\gamma}_{\mathbf{2}}} & & \downarrow & & \\
0 & \longrightarrow & \underbrace{J^2(\hat{Y}_4)}_{\oint C_3 \text{ 'Wilson lines'}} & \longrightarrow & \underbrace{\mathbf{H^4_D}(\hat{\mathbf{Y}}_{\mathbf{4}}, \mathbb{Z}(\mathbf{2}))}_{\textbf{full gauge data}} & \xrightarrow{\hat{c}_2} & \underbrace{H^{2,2}_{\mathbb{Z}}(\hat{Y}_4)}_{\text{field strength } G_4} & \longrightarrow & 0
\end{array}$$

Properties of $\hat{\gamma}_2$ of importance for us:

1. Changing cycles up to rational equiv. does not change gauge data!
2. Combining with $\hat{c}_2$ gives flux: $G_4(A) = \hat{c}_2 \circ \hat{\gamma}_2(A) \in H^{2,2}_{\mathbb{Z}}(\hat{Y}_4)$

Gauge background in F/M-theory

$$
\begin{array}{ccccccccc}
 & & & & \overbrace{\mathbf{CH^2(\hat{Y}_4)}}^{\textbf{geometry}} & & & & \\
0 & \longrightarrow & \mathrm{CH}^2_{\mathrm{hom}}(\hat{Y}_4) & \longrightarrow & \mathbf{CH^2(\hat{Y}_4)} & \xrightarrow{\gamma_2} & H^{2,2}_{alg}(\hat{Y}_4) & \longrightarrow & 0 \\
 & & \downarrow{\scriptstyle AJ} & & \downarrow{\scriptstyle \hat{\gamma}_2} & & \downarrow & & \\
0 & \longrightarrow & \underbrace{J^2(\hat{Y}_4)}_{\oint C_3\ '\text{Wilson lines}'} & \longrightarrow & \underbrace{\mathbf{H^4_D(\hat{Y}_4, \mathbb{Z}(2))}}_{\textbf{full gauge data}} & \xrightarrow{\hat{c}_2} & \underbrace{H^{2,2}_{\mathbb{Z}}(\hat{Y}_4)}_{\text{field strength } G_4} & \longrightarrow & 0
\end{array}
$$

Further properties:

- $\hat{\gamma}_2$ is surjective (over $\mathbb{Q}$) iff the Hodge conjecture holds
- $\hat{\gamma}_2$ is in general not injective, i.e. different elements in $\mathrm{CH}^2(\hat{Y}_4)$ may give same gauge data.

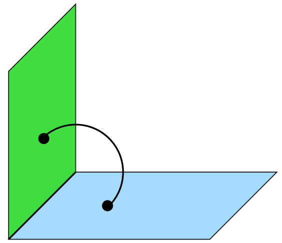
Step 2:
Extracting the cohomologies

Local picture

Local approach: Twisted Theory on 7-brane Σ_i in F-theory

[Beasley,Heckman,Vafa'08] [Donagi,Wijnholt'08]

- 7-branes intersecting over curve $C_{ab} = \Sigma_a \cap \Sigma_b$



- Massless $\mathcal{N} = 1$ chiral multiplets on C_{ab}:
  couple to gauge bundle $L_{ab} = L_a \otimes L_b^*|_{C_{ab}}$

  chiral multiplet in R_{ab} $\qquad H^0(C_{ab}, L_{ab} \otimes \sqrt{K_{C_{ab}}})$
  anti-chiral multiplet in R_{ab} $\qquad H^1(C_{ab}, L_{ab} \otimes \sqrt{K_{C_{ab}}})$

  [Katz,Sharpe'02] [Beasley,Heckman,Vafa'08] [Donagi,Wijnholt'08]

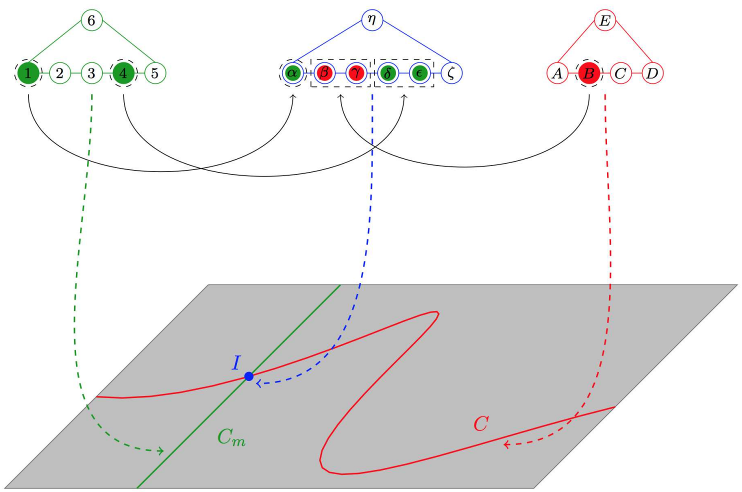
  $\sqrt{K_{C_{ab}}}$: spin bundle induced by holomorphic embedding of C_{ab} cf.

  [Intriligator,Jockers,Mayr,Morrison,Plesser'12]

- chiral index: $\chi(R_{ab}) = \int_{C_{ab}} c_1(L_{ab})$

A cohomology formula

Match this to global data: [Bies,Mayrhofer,Pehle,TW'14] [Bies,Mayrhofer,TW'17]



- Fix element $A \in \mathrm{CH}^2(\hat{Y}_4)$ with $[A] = G_4 \in H^{2,2}(\hat{Y}_4)$
- Matter surface $\pi_{\mathbf{R}} : S^a(\mathbf{R}) \to C_{\mathbf{R}}$
- Intersect $S^a(\mathbf{R}) \cap A \in \mathrm{CH}^2(S^a(\mathbf{R}))$
- Projection to base B_3 gives points on matter curve C_R:

  $$A|_{\mathbf{R}} := \pi_{\mathbf{R}*}(S^a(\mathbf{R}) \cap A) \in \mathrm{CH}^1(C_{\mathbf{R}}) \cong \mathrm{Pic}(C_{\mathbf{R}}) \Rightarrow L_{\mathbf{R}} = \mathcal{O}_{C_{\mathbf{R}}}(A|_{\mathbf{R}})$$

- Massless $\mathcal{N} = 1$ chiral multiplets counted by [Bies,Mayrhofer,Pehle,TW'14]

  $$H^i(C_R, L_{\mathbf{R}} \otimes \sqrt{K_{C_{\mathbf{R}}}}), \qquad i = 0, 1$$

✓ chiral index $\chi(\mathbf{R}) = \deg(L_{\mathbf{R}}) = [S^a(\mathbf{R})] \cdot [A] \equiv \int_{S^a(\mathbf{R})} G_4$

A cohomology formula

Justification:

- Zero modes in $\beta^a(\mathbf{R})$ come from quantization of the moduli space of M2 wrapped on fibre of $S^a(\mathbf{R})$ [Witten'97]
- Gauge background on $C_{\mathbf{R}}$ from integrating C_3 over fibre is mathematically given precisely by operation of intersection and projection

$$A|_{\mathbf{R}} := \pi_{\mathbf{R}*}(S^a(\mathbf{R}) \cap A)$$

- All operations are compatible with rational equivalence and hence, by the refined cycle map, with the definition of the gauge background.

Good to know:

Use rational equivalence relations to transform intersection into transverse one

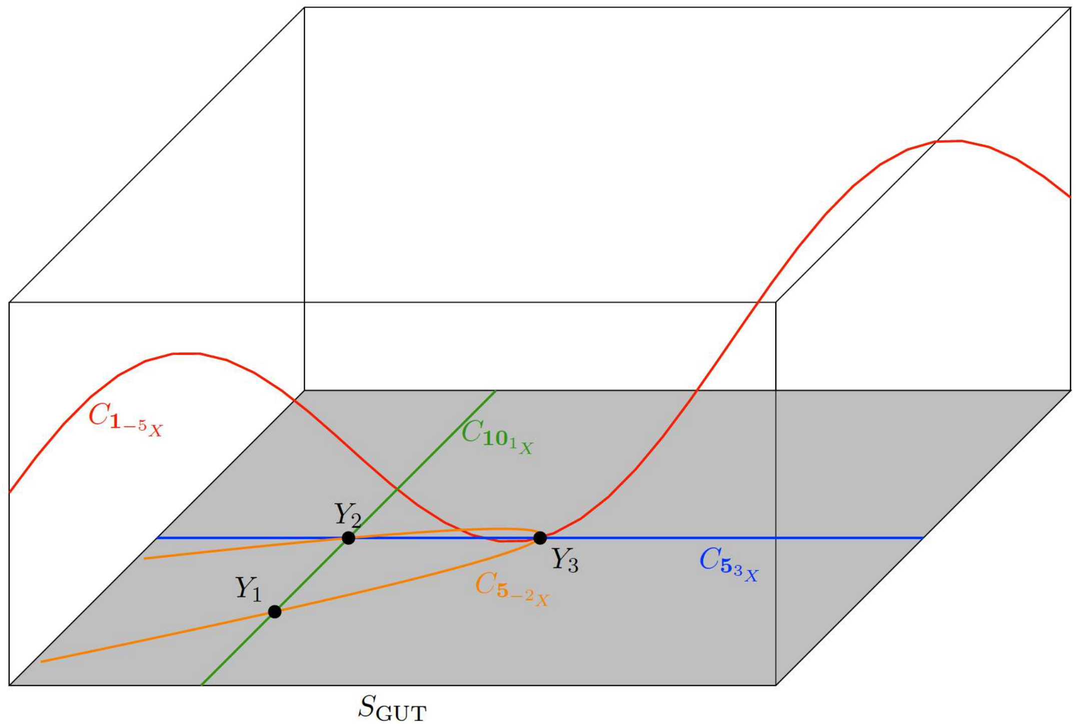
Step 3:
Computing the cohomologies

See Talk by Martin Bies

General idea:

Translate points on $C_{\mathbf{R}}$ into ideal sheaf and compute its extension groups

Explicit Example



Gauge group $SU(5) \times U(1)_X$

- Explicit fibration constructed in [Mayrhofer,Krause,TW'11&'12]
- 4 types of charged matter:

  $\mathbf{10}_1$ $\mathbf{5}_3$ $\mathbf{5}_{-2}$ $\mathbf{1}_5$
- Relevant intersection points = Yukawa points!

Available vertical fluxes: [Bies,Mayrhofer,TW'17]

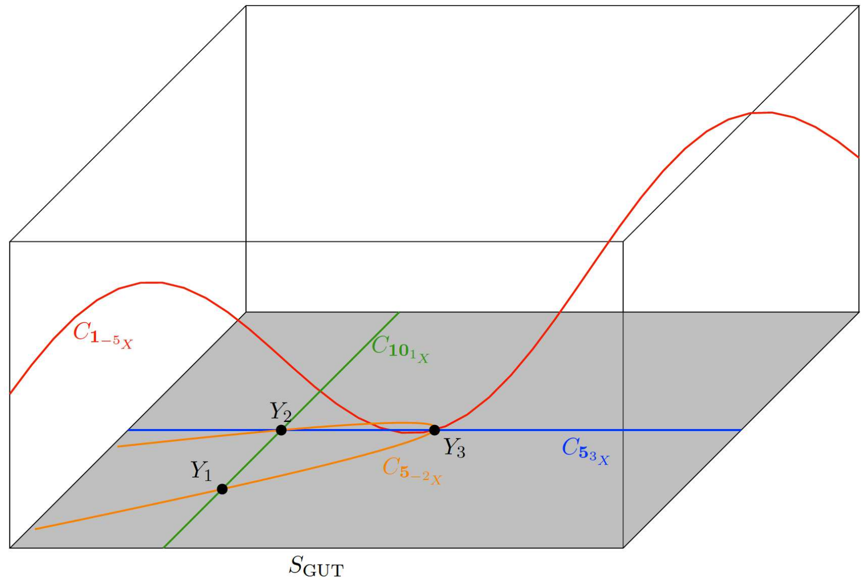
- $U(1)_X$ flux: $A_X(F) = F \cap \mathrm{w}_X$
- Matter surface induced fluxes: $A(\mathbf{10}_1)$, $A(\mathbf{5}_3)$, $A(\mathbf{5}_{-2})$, $A(\mathbf{1}_5)$

Only two independent ones due to 3 relations within the Chow ring

$\leftrightarrow$ anomaly cancellation in the Chow ring! cf. [Lin,TW'16] [Bies,Mayrhofer,TW,17]

Explicit Example

Example: Flux $A(\mathbf{10}_1)$



1) Transverse Intersection with matter surfaces $S(\mathbf{5}_3)$, $S(\mathbf{5}_{-2})$, $S(\mathbf{1}_5)$

- $\pi_{C*}(A(\mathbf{10}_{1_X}) \cap S(\mathbf{5}_{3_X})) = 2Y_2$
- $\pi_{C*}(A(\mathbf{10}_{1_X}) \cap S(\mathbf{5}_{-2_X})) = 2Y_2 - 3Y_1$
- $\pi_{C*}(A(\mathbf{10}_{1_X}) \cap S(\mathbf{1}_{5_X})) = 0$

2) Non-transverse intersection with matter surface $S(\mathbf{10}_{1_X})$:

Use relation $A(\mathbf{10}_{1_X}) = -A(\mathbf{5}_{3_X}) - A(\mathbf{5}_{-2_X})$ to rewrite transversely

- $\pi_{C*}(A(\mathbf{10}_{1_X}) \cap S(\mathbf{10}_{1_X})) = 3Y_1 - 4Y_2$

Explicit Example

Similar computations possible for second independent flux $A_X(F)$

Putting everything together

1. Build general linear combination satisfying quantization condition:
   - $A_{\rm tot} = \lambda A(\mathbf{10}_{1_X}) + A_X(F)$
   - Associated $G_4 = [A_{\rm tot}]$:

   $$G_4 + \frac{1}{2} c_2(\hat{Y}_4) \in H^4(\hat{Y}_4, \mathbb{Z})$$

   subject to further tadpole and D-term constraints in fully consistent model
2. Massless matter: $H^i(C_{\mathbf{R}}, \mathcal{L}_{\mathbf{R}})$ $\qquad \mathcal{L}_{\mathbf{R}} = \mathcal{O}_{C_{\mathbf{R}}}(A_{\rm tot}|_{C_{\mathbf{R}}}) \otimes \sqrt{K_{C_{\mathbf{R}}}}$
3. $\mathcal{L}_{\mathbf{R}}$ on $C_{\mathbf{R}}$ $=$ sheaf on the base B_3 (specified by vanishing ideal)

   If B_3 is toric, or embedded into a toric space X_Σ, the sheaf cohomology groups can be computed via computer algebra *gap* following general algorithms provided by [Barakat,Lange-Hegermann'10-'17]

Explicit example

Base space $B = \mathbb{P}^3$ with hyperplane class H

$A_{\rm tot} = A(\mathbf{10}_{1_X}) + A_X(\frac{1}{2}H)$ is well-quantised

- $\mathcal{L}_{\mathbf{10}_{1_X}} = \mathcal{O}_{\mathbb{P}^2}(-18)|_{C_{\mathbf{10}_{1_X}}}$ $\quad$ $\mathcal{L}_{\mathbf{5}_{3_X}} = \mathcal{O}_{\mathbb{P}^2}(13)|_{C_{\mathbf{5}_{3_X}}}$
- $\mathcal{L}_{\mathbf{5}_{-2_X}} = \mathcal{O}_{C_{\mathbf{5}_{-2_X}}}(-5Y_1) \otimes \mathcal{O}_{\mathbb{P}^2}(14)|_{C_{\mathbf{5}_{-2_X}}}$

Explicit evaluation of cohomologies depends on complex structure moduli - i.e. specific form of matter curves

Example: Tate coefficients $a_i = a_{i,j} w^j$ $w = 0$: SU(5) brane

$$a_{1,0} = c_1 (x_1 - x_2)^4 , \qquad a_{2,1} = c_2 x_1^7, \qquad a_{3,2} = c_3 x_2^{10}, \qquad a_{4,3} = c_4 x_3^{13}.$$

$$h^i \left(C_{\mathbf{10}_{1_X}}, \mathcal{L}_{\mathbf{10}_{1_X}} \right) = (0, 74), \qquad h^i \left(C_{\mathbf{5}_{3_X}}, \mathcal{L}_{\mathbf{5}_{3_X}} \right) = (95, 0)$$

$$h^i \left(C_{\mathbf{5}_{-2_X}}, \mathcal{L}_{\mathbf{5}_{-2_X}} \right) = (22, 43)$$

Moduli Dependence

Explicit dependence on complex structure moduli:

$a_{\tilde{1},0}$	$a_{\tilde{2},1}$	$a_{\tilde{3},2}$	$a_{\tilde{4},3}$	$h^i\left(C_{\mathbf{5}_{-2}}, \mathcal{L}\left(A, C_{\mathbf{5}_{-2}}\right)\right)$
$(x_1 - x_2)^4$	x_1^7	x_2^{10}	x_3^{13}	$(22, 43)$
$(x_1 - x_2) \cdot x_3^3$	x_1^7	x_2^{10}	x_3^{13}	$(21, 42)$
x_3^4	x_1^7	$x_2^7 \cdot (x_1 + x_2)^3$	$x_3^{12} \cdot (x_1 - x_2)$	$(11, 32)$
$(x_1 - x_2)^3 \cdot x_3$	x_1^7	x_2^{10}	x_3^{13}	$(9, 30)$
x_3^4	x_1^7	$x_2^8\,(x_1 + x_2)^2$	$x_3^{11} \cdot (x_1 - x_2)^2$	$(7, 28)$
x_3^4	x_1^7	x_2^{10}	$x_3^8 \cdot (x_1 - x_2)^5$	$(6, 27)$
x_3^4	x_1^7	$x_2^9 \cdot (x_1 + x_2)$	$x_3^{10} \cdot (x_1 - x_2)^3$	$(5, 26)$

Anomalies and Algebraic Cycles

Theorem: [Bies,Mayrhofer,TW'17] partly anticipated in [Lin,TW'16]

Absence of gauge and gauge-gravitational anomalies in F-theory compactified on elliptically fibered n-fold $\hat{Y}_n$ *with* $n = 3, 4$ *is equivalent to the following universal relations in cohomology* $H^{2,2}(\hat{Y}_n, \mathbb{R})$*:*

$$\sum_{\mathbf{R}} \sum_{a} \beta^a_\Lambda(\mathbf{R}) \beta^a_\Sigma(\mathbf{R}) \beta^a_\Gamma(\mathbf{R}) \, [S^a_{\mathbf{R}}]_{\rm vert} \quad = \quad 3 \, [F]_{(\Gamma} \cdot [\pi_*(F_\Lambda \cap F_{\Sigma)})]$$

$$\sum_{\mathbf{R}} \sum_{a} \beta^a_\Lambda(\mathbf{R}) \, [S^a_{\mathbf{R}}]_{\rm vert} \quad = \quad -6 \, [\bar{K}] \cdot [F_\Lambda]$$

Λ, Σ, Γ: any (Cartan) $U(1)$ F_Λ: Divisor class such that $C_3 = A_\Lambda \wedge F_\Lambda$

goes beyond intersection theoretic identities of [Grassi,Morrison'11], [Park'12], [Grimm,Klevers,Cvetič'12] cf talk by Grimm

Conjecture:

This holds even at the level of rational cycle classes (i.e. Chow classes)

$\Longrightarrow$ Relations between flux cycles!

Conclusions

Gauge data in global F/M-theory specified via $\mathrm{CH}^2(\hat{Y}_4)$

- Intersection theory modulo rational equivalence extracts line bundle on matter curves L
- Exact massless matter counted by cohomology group $H^i(C, L \otimes \sqrt{K_C})$
- Explicit evaluation in globally consistent geometries via computer algebra

A wealth of applications ahead of us:

- Apply to existing chiral 3-generation models
- Find realistic spectra competitive with heterotic models
- Understand better $H^{2,2}_{\mathrm{rem}}(\hat{Y}_4)$ and $H^{2,2}_{\mathrm{hor}}(\hat{Y}_4)$
- Understand better $\mathrm{CH}^2(\hat{Y}_4)$ and its deviations from $H^{2,2}(\hat{Y}_4)$
- Non-abelian gauge backgrounds?

Construction of fluxes

$G_4 = [A]$ takes values in $H^{2,2}(\hat{Y}_4, \mathbb{R})$ subject to [Witten'96]

$$G_4 + \frac{1}{2} c_2(\hat{Y}_4) \in H^4(\hat{Y}_4, \mathbb{Z})$$

Orthogonal decomposition: [Greene,Morrison,Plesser'92], ...

$$H^{2,2}(\hat{Y}_4, \mathbb{R}) = H^{2,2}_{\text{vert}}(\hat{Y}_4, \mathbb{R}) \oplus H^{2,2}_{\text{hor}}(\hat{Y}_4, \mathbb{R}) \oplus H^{2,2}_{\text{rem}}(\hat{Y}_4, \mathbb{R})$$

- $H^{2,2}_{\text{vert}}(\hat{Y}_4) = H^{1,1}(\hat{Y}_4) \wedge H^{1,1}(\hat{Y}_4)$
- $H^{2,2}_{\text{hor}}(\hat{Y}_4) \leftrightarrow$ by variation of Hodge structure from $H^{4,0}(\hat{Y}_4)$
- $H^{2,2}_{\text{rem}}(\hat{Y}_4)$: remainder [Watari,Braun'14]

In all many geometries matter surface classes are in $H^{2,2}_{\text{vert}}(\hat{Y}_4)$

$\Rightarrow$ only $G_4 \in H^{2,2}_{\text{vert}}(\hat{Y}_4)$ gives chiral spectrum here

$\Rightarrow$ For this talk focus on $G_4 \in H^{2,2}_{\text{vert}}(\hat{Y}_4)$

Construction of fluxes

Fluxes G_4 in F-theory subject to transversality conditions

$$G_4 \cdot \underbrace{[S_0]}_{\text{zero-section}} \cdot \underbrace{[D^{\mathrm{b}}_\alpha]}_{\text{base divisor}} = 0, \qquad G_4 \cdot [D^{\mathrm{b}}_\alpha] \cdot [D^{\mathrm{b}}_\beta] = 0$$

Gauge invariance requires

$$G_4 \cdot \underbrace{[E_i]}_{\text{resol. div.}} \cdot [D^{\mathrm{b}}_\alpha] = 0$$

Two types of data with $G_4 \in H^{2,2}_{\mathrm{vert}}(\hat{Y}_4)$:

1) U(1) flux in presence of extra section:

- Expanding 3-form $C_3 = \mathbb{A}_X \wedge [\mathsf{w}_X]$ gives $U(1)_X$ potential $\mathbb{A}_X$
- $G_4 = [F] \wedge [\mathsf{w}_X] \quad [F] \in H^{1,1}(B_3), \quad [\mathsf{w}_X] \in H^{1,1}(\hat{Y}_4)$
- Underlying algebraic cycle:

$$A_X = F \cap \mathsf{w}_X \in \mathrm{CH}^2(\hat{Y}_4) \qquad F \in \mathrm{CH}^1(B_3) \qquad \mathsf{w}_X \in \mathrm{CH}^1(\hat{Y}_4)$$

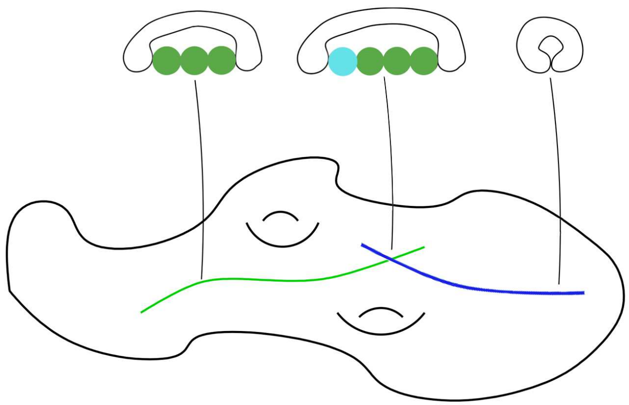
Construction of fluxes

2) Matter surface flux

Representation $\mathbf{R} \leftrightarrow$ states/weights labeled by $a = 1, \ldots, \dim(\mathbf{R})$

$S^a(\mathbf{R})$: linear combination of $\mathbb{P}^1$s fibered over curve $C_\mathbf{R}$

$\Rightarrow$ element in $\mathrm{CH}^2(\hat{Y}_4)$!



Gauge group invariance requires: [Borchmann,Mayrhofer,Palti,TW'13]

$$\begin{aligned} A^a(\mathbf{R}) &= S^a(\mathbf{R}) + \Delta^a(\mathbf{R}) \in \mathrm{CH}^2(\hat{Y}_4) \\ \Delta^a(\mathbf{R}) &= -\left(\beta^a(\mathbf{R})_i^T C_{ij}^{-1}\right) E_j|_{C_\mathbf{R}} \in \mathrm{CH}^2(\hat{Y}_4) \end{aligned}$$

Turns out: $A^a(\mathbf{R}) \equiv A(\mathbf{R})$ for all a

Can explicitly perform intersection with matter surfaces and extract bundle data for cohomologies!