An exploration of threefold bases in F-theory
1510.04978 & upcoming work with W. Taylor

Yi-Nan Wang

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F-theory landscape program

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An exploration of threefold bases in F-theory
Classify distinct F-theory compactifications to 4D

F-theory compactification on an elliptic CY4 $M$, with complex threefold base $B$. 

Our goal: explore large sets of (compact, smooth) bases; Characterize, Classify, Count.
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Study the “non-Higgsable phase”, where the gauge groups on the base are minimal.
Characterization of bases

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In the Weierstrass form:

\[ y^2 = x^3 + fx + g, \quad (1) \]

\( f \) and \( g \) are taken to be generic sections of \( \mathcal{O}(-4K_B) \), \( \mathcal{O}(-6K_B) \). They are polynomials with generic random coefficients, such that the discriminant \( \Delta \) vanish to lowest order over any locus.
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Another property: the number of complex structure moduli \( h^{3,1} \) of the elliptic CY4 is maximal.
Classification of 2D bases

• Minimal model program of complex surfaces: Enriques-Kodaira classification.

• Bases for elliptic CY3: rational surface & Enrique surface (Grassi 91’).

Classify rational surface $B$ which can be a base of elliptic CY3 used in F-theory: Consequently blowing up $P^2$ and Hirzebruch surfaces $F_0, \ldots, F_{12}$.

Condition: In the generic fibration, $(f, g)$ does not vanish to order $(4, 6)$ or higher on any cod-1 or cod-2 locus on $B$.

• Almost done: (Morrison, Taylor 12'; Martini, Taylor 14'; Taylor, YNW 15').
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Toric threefolds

Gluing $\mathbb{C}^3$ together such that there is an action of complex torus $(\mathbb{C}^*)^3$. 

Description: a fan in the lattice $\mathbb{Z}^3$: $\Sigma$ with set of 3D, 2D, 1D cones.

- **1D ray:** $v_i$ corresponds to divisor $D_i$; $z_i = 0$. 
  
  $N(v_i) = h_1(B_i) + 3$.

- **2D cone:** $v_i v_j$ corresponds to curve $z_i = z_j = 0$.

- **3D cone:** $v_i v_j v_k$ corresponds to point $z_i = z_j = z_k = 0$. 

$(0,0,1)$ $(1,0,0)$ $(0,1,0)$ $(-1,-1,-1)$

$z_1 = 0$ $z_2 = 0$ $z_3 = 0$ $z_4 = 0$

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Generators of holomorphic section $m_p$ of line bundle $L = \sum_i a_i D_i \Leftrightarrow$ points $p$ in the dual lattice $\mathbb{Z}^3$:

$$\{p \in \mathbb{Z}^3, \forall v_i, \langle p, v_i \rangle \geq -a_i \}.$$  \hspace{1cm} (2)

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Anti-canonical bundle $-K_B = \sum_i D_i$. Hence $f$ and $g$ are linear combinations of monomials in set $\mathcal{F}$ and $\mathcal{G}$:

$$\mathcal{F} = \{p \in \mathbb{Z}^3, \forall v_i, \langle p, v_i \rangle \geq -4\}. \quad (4)$$

$$\mathcal{G} = \{p \in \mathbb{Z}^3, \forall v_i, \langle p, v_i \rangle \geq -6\}. \quad (5)$$
(1) Blow up a point $v_i v_j v_k$: add another ray $\tilde{v} = v_i + v_j + v_k$.
(2) Blow up a curve $v_i v_j$: add another ray $\tilde{v} = v_i + v_j$. 
Blow up/down toric threefolds

1. Blow up a point $v_i v_j v_k$: add another ray $\tilde{v} = v_i + v_j + v_k$.
2. Blow up a curve $v_i v_j$: add another ray $\tilde{v} = v_i + v_j$.

- The set $\mathcal{F} \& \mathcal{G}$ after the blow up is a subset of the previous ones.
- Blow up (4,6) curve does not change the set $\mathcal{F} \& \mathcal{G}$.
Random walk on the toric threefold landscape

- Start from \( \mathbb{P}^3 \), do a random sequence of 100,000 blow up/downs.
- Never pass through bases with cod-1 or cod-2 (4,6) singularities (excluding \( E_8 \) gauge group).
- In total 100 runs. \( h^{1,1}(B) = 1 \sim 120 \).
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<table>
<thead>
<tr>
<th>Group</th>
<th>SU(2)</th>
<th>SU(3)</th>
<th>$G_2$</th>
<th>SO(7)</th>
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<td>2.0</td>
<td>9.7</td>
<td>$4 \times 10^{-6}$</td>
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<td></td>
<td>1.0</td>
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</tbody>
</table>

Average number of non-Higgsable gauge group on a base.

- 76% of bases have $SU(3) \times SU(2)$ non-Higgsable cluster.
Estimation of the number of distinct bases

- Do a limited random walk with cap $h^{1,1}(B) \leq 7$, get the number of bases $N(7)$ and $N(2)$.
- We know there is 1 base with $h^{1,1}(B) = 1$: $\mathbb{P}^3$. 27 bases with $h^{1,1}(B) = 27$.
- The number of bases with $h^{1,1}(B) = 7$ is about $27 \times N(7)/N(2)$.
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Total number $\sim 10^{48}$.
New Monte Carlo approach

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For the case of surface base/elliptic CY3, this exclude a large region.
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We call the base without cod-2 (4,6) locus a good base.

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New Monte Carlo approach

- In this approach, we cannot perform a random walk, because the good base is extremely rare among resolvable bases.
- Instead, we do a sequence of blow ups starting from a single base, e.g. $P_3$, until hitting the end point where one cannot blow up to get a resolvable base.
- According to the definition, the end point is always good. But most of the bases between $h_1$, $1_{1B}$, $1_{B}$ are only resolvable.
- Assign weight factor to each base on each sequence to compute the total number of resolvable/good bases with each $h_{1_{1B}}$. $B$. 

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(3) We can consider more general starting point bases with non-Higgsable clusters.
New results

- The distribution of resolvable bases centralized at very large $h^{1,1} \sim 4,000$.
- The total number of resolvable bases $\sim 10^{1,700}$, bigger than the number $10^{755}$ in 1706.02299 by Halverson, Long and Sung.
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- Autocracy. Similar story happens in the flux vacua story (YNW, Taylor 15’), where one geometry with $10^{272,000}$ flux vacua dominates.
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• The gauge groups are almost always $E_8^a \times F_4^b \times G_2^c \times SU(2)^d$. $SU(3)$ and $SO(8)$ seldom appears.

After computing $h_{1,1}(X)$, $h_{3,1}(X)$ over the end point bases $B$, we found that they resemble the mirror of simple elliptic CY4s over simple bases.

(1) For the bases with $h_{1,1}(B) = 2303$, $h_{1,1}(X) = 3878$, $h_{3,1}(X) = 2$: mirror of generic elliptic CY4 over $P^3$.

(2) For the bases with $h_{1,1}(B) = 2591$, $h_{1,1}(X) = 4358$, $h_{3,1}(X) = 3$: mirror of generic elliptic CY4 over generalized Hirzebruch threefold $\tilde{F}_3$.

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