

Where in F-theory is the supersymmetric standard model?

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Washington (Wati) Taylor, MIT

Based in part on papers written in collaboration with various subsets of:

L. Anderson, A. Grassi, J. Gray, J. Halverson, S. Johnson,
D. Klevers, D. Morrison, D. Park, [N. Raghuram](#), J. Shaneson, [Y. Wang](#)

Challenge: connect string theory \leftrightarrow observable physics

Traditional approach:

- Pick compactification scheme (IIB + flux, heterotic, IBM, F-theory, ...)
- Assemble ingredients for $SU(3) \times SU(2) \times U(1)$, maybe $SU(5)$ etc.
- Fine tune to get details of standard model

Issues:

- 1) Generally no global picture for context:
How typical or unusual is SM in space of theories?
- 2) In most approaches: starting assumption is “blank slate”
Assumes typical vacuum has no gauge fields or matter

F-theory has unique power:

- Holomorphy and complex geometry \Rightarrow global picture of {vacua}
- Generic vacua have nontrivial gauge group G everywhere in moduli space
- Some G natural (e.g. $E_6, E_7, E_8, SU(3) \times SU(2)$), others not ($SU(5)$)
- “Hidden” sectors $\Rightarrow \sim$ dark matter
- Realistic framework for statistical study of {vacua}
 - \rightarrow What G , matter are “natural”
 - \rightarrow How can SM arise? Is it natural or requires extreme tuning?
[cf. also Halverson, Nelson talks]

Caveat: current story requires/assumes SUSY

Research program goals:

Study global { F-theory vacua } (w/SUSY in 6D, 4D)

- Match with allowed { SUGRA (+ YM, matter) } (close match in 6D)
- What G , matter are possible?
What G , matter are typical?
- How can 4D Standard Model arise? Is it natural?

Big questions but surprising progress in last 8 years.

F-theory:

IIB on base B_2 (6D), B_3 (4D)

Axiodilaton $\tau \rightarrow$ ell. curve modulus

Physics encoded in Weierstrass model:

$$y^2 = x^3 + fx + g$$

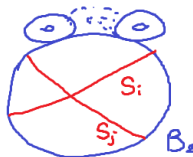
Codim. 1 singularities (S_i): gauge group,

Codim. 2: matter [cf. Raghuram talk]

To study { F-theory vacua }:

- **Classify B' 's**: generic fibration typically has non-Higgsable G
 (e.g. -3 curve in $B_2 \rightarrow \text{SU}(3)$)
- **Classify tunings/Weierstrass models**

Finite # B' 's, top. distinct CY3, CY4's [rigorous for 6D, pieces for 4D]



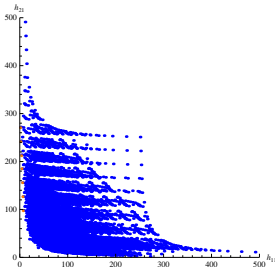
Summary of 6D story: [single theory: connected moduli space]

Bases: blow-ups of \mathbb{P}^2 , \mathbb{F}_m ($m \leq 12$), Enriques [Grassi]

Classify bases: 61,539 toric [Morrison/WT];
 non-toric: all w/ $h^{2,1}(X) \geq 150$ [WT/Wang]

Classify tunings [Johnson/WT, Huang/WT]

- All known Hodge #'s $\geq 240 \Leftarrow$ EF
- All but 10 (dP) bases \Rightarrow non-Higgsable G
- Typical non-Higgsable G , matter: e.g. $(G_2 \times SU(2))^4 \times F_4 \times E_8^2 \times E_6$
- Max $h^{2,1}$: EF on \mathbb{F}_{12} : $(h^{1,1}, h^{2,1}) = (11, 491)$

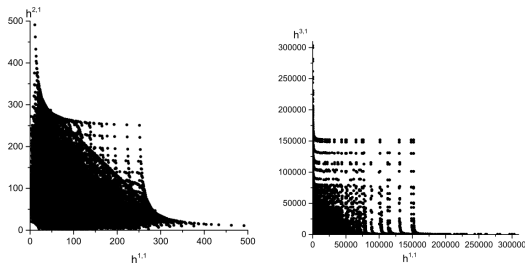


— Good match w/allowed 6D SUGRA, some open issues (U(1)'s, ...)

— Good global picture of 6D F-theory models: most questions answerable

4D F-theory compactifications: Story parallel in many ways

- Compactify on elliptic Calabi-Yau fourfold, base $B_3 =$ complex threefold
 Most known CY4's elliptic (cf. [Gray/Haupt/Lukas, Anderson/Gao/Gray/Lee])
- Empirical data suggest similar structure (though less complete for CY4's)



4D theories significantly more subtle: [cf. eg Grimm talk]

- Minimal models (Mori theory) more subtle
- F-theory $\subset \mathcal{V}_4$ (e.g. heterotic on quintic)
- Fluxes, superpotential, seven-brane dynamics not completely understood

But evidence so far: moduli space of CY4 geometries parallel to CY3 story

4D F-theory models: Classifying bases [cf. Halverson, Wang talks]

Harder: # B_3 very large

Toric MC (no codim. 2 (4, 6), E_8, \dots)

$$\Rightarrow |\{B_3\}| > \sim 10^{50} \text{ [WT/Wang]}$$

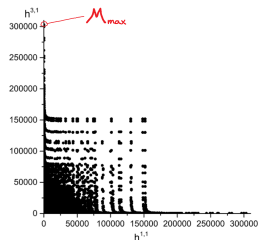
Trees on Fanos (w/ codim. 2 (4, 6))

$$\Rightarrow |\{B_3\}| > \sim 10^{750} \text{ [Halverson/Long]}$$

One-way MC [WT/Wang]

$$|\{B_3\}| > \sim 10^{1680} \text{ (w/ codim. 2 (4, 6))}$$

$$|\{B_3\}| > \sim 10^{240} \text{ (w/o codim. 2 (4, 6))}$$



- Essentially all $B_3 \Rightarrow$ geometrically non-Higgsable G
- Maximum $h^{3,1}$: $\mathcal{M}_{max}, (h^{1,1}; h^{3,1}) = (252; 303, 148)$

4D F-theory geometry: tuning vs. generic G

Non-Higgsable clusters \sim 6D: divisors (surfaces) w/ negative normal bundles
 [Anderson/WT, Grassi/Halverson/Shaneson/WT, Morrison/WT]

Geometrically non-Higgsable G :

Factors : $SU(2)$, $SU(3)$, G_2 , $SO(7)$, $SO(8)$, F_4 , E_6 , E_7 , E_8 (no $SU(5)$, $SO(10)$)

Local products w/matter:

$G_2 \times SU(2)$, $SO(7) \times SU(2)$, $SU(2) \times SU(2)$, $SU(3) \times SU(2)$, $SU(3) \times SU(3)$

6D: protected by D-terms; 4D, F terms also likely relevant.

NH U(1) factors: possible but rare [6D: Martini/WT, Morrison/Park/WT; 4D: Wang]

Geometrically possible w/tuning: all simple G , U(1) products

Upper bounds on G from geometry

e.g. \mathbb{P}^3 (likely max): $SU(N)$, $SO(N)$, $N \leq 32$; $Sp(N)$, $N \leq 16$

Max G : likely $E_8^{2561} \times F_4^{7576} \times G_2^{30168} \times SU(2)^{30200}$ [Candelas/Perevalov/Rajesh]

Caveat: this is geometry; fluxes, superpotential may modify G up or down.

Codimension two: matter [cf. Raghuram talk]

“Generic” matter representations:

Fundamental, antisymmetric, adjoint of $SU(N)$, etc.:

Arise in standard Tate tunings of Weierstrass models.

F-theory appears to strongly limit possible representations of nonabelian G :

$SU(N)$:

3-index antisymmetric representations of $SU(6)$, $SU(7)$, $SU(8)$, similar for Sp

2-index symmetric representation for $SU(N)$

$SU(2)$: 3-index symmetric representation

Latter representations require non-UFD on singular divisors;

appears no other exotic representations possible without (4, 6) cod. 2

Classifying and constructing possible general $U(1)$ charges: current research

Discrete gauge factors, torsion: even less understood [cf. Cvetic talk]

“Typical” vacua (assume toric base)

Typical toric base has e.g. $E_8^a \times F_4^b \times G_2^c \times SU(2)^d$ [cf. Wang talk]

Toric: $G_2 \times SU(2)$ has matter on \mathbb{P}^1 toric curve; no matter w/o fluxes
 [cf. Weigand talk]

Flux vacua:

Conventional wisdom (Ashok-Denef-Douglas):

\Rightarrow in regime $h^{1,1} \ll h^{3,1}$, #vacua $N(X) \sim 10^{0.9 h^{3,1}(X)}$

\mathcal{M}_{\max} is elliptically fibered; B_2 over \mathbb{P}^1 . Dominates set of flux vacua?
 [WT/Wang]

$N(\mathcal{M}_{\max}) \sim 10^{272,000}$ non-Higgsable $G_{\max} = E_8^9 \times F_4^8 \times (G_2 \times SU(2))^8$

Circumstantial evidence: $\sum_{X \neq \mathcal{M}_{\max}} N(X) < 10^{-3000} N(\mathcal{M}_{\max})$

Disconnected non-Higgsable G components \Rightarrow dark matter?

How can we realize the supersymmetric standard model in F-theory?

- Tune SU(5) GUT** (*a la* [Beasley/Heckman/Vafa, Donagi/Wijnholt])
 (or tuning $SU(3) \times SU(2) \times U(1)$ (*e.g.*, [Lin/Weigand]))

 - **Can't be done on \mathcal{M}_{\max}**
 - **Requires tuning many moduli** [Braun/Watari, Halverson/Tian]
 - Flux breaking **needs special (*e.g.* non-toric) divisors; Yukawas, ...**
 [Heckman/Morrison/Vafa, Marsano/Saulina/Schafer-Nameki]
- Use non-Higgsable $SU(3) \times SU(2)$** [Grassi/Halverson/Shaneson/WT]

 - **NHC's fairly natural** ($\sim 10\%$ of products, $\sim 2/\text{base}$ from MC w/Wang)
 - **Can't be done on \mathcal{M}_{\max}**
 - **Tuning the U(1) may be expensive** ($\sim SU(2)$ on $-K + X_{\text{eff}}$, breaking adjoint)
- GUT breaking through 7-brane flux on non-Higgsable E_6, E_7, E_8**

 - **Doesn't seem to work on \mathcal{M}_{\max}**
 (E_8 's all on $D = \mathbb{F}_m$, no Yukawas [Beasley/Heckman/Vafa])
 - **Need exotic (*e.g.* non-toric) local structure.**

Problem: Appears \mathcal{M}_{\max} dominates flux vacua but apparently can't give SSM.

How to get around this?

- **Non-toric bases?** [work w/ Wang, Morrison]

Preliminary analysis suggests possible that $|\{B_3^{\text{non-toric}}\}| > \sim 10^{300,000}$.

If so, could dominate landscape, make SSM natural
 (currently my favorite solution)

Otherwise, must give something up

- **Give up SUSY breaking at sub-Planck scale?** [cf. Abel talk]
 Possible, but makes life very hard for string theorists!
- Maybe $10^{300,000}$ vacua from some other approach? G_2 ? Non-geometric?

Are NHC's so generic that they may also be typical behavior for other vacuum constructions? Without SUSY?

Or: maybe we are thinking about something wrong.

- Perhaps ADD estimate doesn't correctly count F-theory flux vacua?
(e.g. measure factor, etc.)
- Perhaps nonperturbative effects produce Yukawas for $E_8 \subset G_{\mathcal{M}_{\max}}$?
- Maybe many fluxes naturally force SU(5)
or a U(1) for non-Higgsable $SU(3) \times SU(2)$?

If we find a natural SSM solution (e.g. non-toric B_3):

⇒ Can study statistics of vacua

⇒ Likely SSM appears in a peaky statistical ensemble

⇒ Could predict correlated features with high probability

Need better handle on interplay of fluxes, NHC's, 7-brane DOF

[current work w/ Anderson, Gray, Halverson, Long; cf. Wiegand talk]

Conclusions

- We have a good handle on the classification of elliptic Calabi-Yau threefolds
- A plausible “bird’s-eye” picture of the global space of elliptic CY4’s (with much work remaining to be done)
- Significant questions regarding the connection of geometry and physics in 4D
- Several ways of realizing $SU(3) \times SU(2) \times U(1)$ in F-theory
- Some sense of what may be more or less natural in 4D F-theory landscape
- Conventional wisdom seems to make SSM unnatural
But non-toric geometry has the potential to save the day!