Phases of Axion Inflation

Wieland Staessens (JdC)

based on 17xx.xxxxx (1503.01015, 1503.02965 [hep-th])

with G. Shiu

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Madrid

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Inflationary axions & String Theory

- Controlled perturbative corrections to V_{inf} Linde (1988) for axions with $a \to a + \varepsilon$:
  $\Rightarrow$ prerequisite for large field inflation (LFI)
  $\rightsquigarrow$ natural inflation Freese-Frieman-Olinto (1990) with $V = \Lambda_s^4 \left[1 - \cos\left(\frac{a}{f}\right)\right]$
  slow-roll for $f > M_{Pl}$ and $V^{1/4} \sim \Lambda_s \sim 10^{16}$ GeV

In StringPheno, we ask questions like:

(1) How to embed LFI?

- ⋆ Multi-axions $\rightsquigarrow$ N-flation (2005), aligned natural (2004), kinetic mixing (McAllister) (2015)
- ⋆ different mass mechanism $\rightsquigarrow$ axion monodromy (2008, 2014)

(2) How are models constrained by quantum gravity?

- ⋆ $f > M_{Pl}$ No-go theorems for single axion Banks et al (2003), Svrček-Witten (2006)
- ⋆ WGC + swampland conjectures (Shiu, Heidenreich, Rudelius, Valenzuela, Blumenhagen, Ooguri, Hebecker, Wolf, Kläwer, Herraez, Montero, Stout)

Here: Explore richness of EFT for stringy axions in Type II with D-branes & mass generation mechanisms

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Stringy Axions from Type II compactifications

reviews: Baumann (2009), Baumann-McAllister (2009,2014), Westphal (2014), Grimm-Louis (2005), Grimm-Lopes (2011), Kerstan-Weigand (2011), Grimm-Louis (2004), Jockers-Louis (2005), Haack-Krefl-Lust-Van Proeyen-Zagermann (2006)

- Closed string axions a^i from dim. red. of p-forms $C_{(p)}$ on $\mathcal{M}_{1,3} \times \mathcal{X}_6/\Omega\mathcal{R}$ ($C_{(p)} \in$ RR-forms + NS 2-form in Type II)

$$a^i \equiv (2\pi)^{-1} \int_{\Sigma^i} C_{(p)}, \qquad p-\text{cycle } \Sigma^i \subset \mathcal{X}_6, \qquad i \in \{1, \ldots, \begin{matrix} h_{11} \\ h_{21}+1 \end{matrix} \}$$

Kinetic terms for p-forms $C_{(p)} \in \rightsquigarrow$ kinetic terms for a^i

- Stack of N D-branes on $\mathcal{M}_{1,3} \times \Sigma^i \rightsquigarrow U(N)$ gauge theory in $4+p$ dim
  Reduction of D-brane CS-action $\rightsquigarrow$ couplings for a^i
  - IIA
    - ⋆ $C_3 \wedge \text{Tr}(G \wedge G) \rightarrow$ anomal. coupling $a^i\, Tr(G \wedge G)$
    - ⋆ $C_5 \wedge F \rightarrow$ Stückelberg-coupling $(da^i - k^i A)^2$ under U(1)
- Subset of axions are eaten by anomalous $U(1)$'s $\rightsquigarrow$ survive as global symmetries
- Also possible in IIB for $C_{2,4}$ using D7-branes with magn. fluxes

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Effective Action & Effective Decay Constant

w/ Shiu-Ye 1503.01015, 1503.02965 [hep-th]

- Type II String Theory compactifications w/ D-branes
  $\rightsquigarrow$ 4d EFT with mixing axions

$$\mathcal{S}^{\text{eff}}_{axion} = \int \left[\frac{1}{2} \sum_{i,j=1}^{N} \mathcal{G}_{ij} (\mathrm{d}a^i - k^i A) \wedge \star_4 (\mathrm{d}a^j - k^j A) - \frac{1}{8\pi^2} \left(\sum_{i=1}^{N} r_i a^i \right) \mathrm{Tr}(G \wedge G) + \mathcal{L}_{gauge} \right]$$

metric mixing ($\mathcal{G}_{ij}$); $U(1)$ mixing $k^i \neq 0$; anomalous coupling

- Diagonalisation of kinetic and potential terms
  $\Rightarrow$ effective decay constant f_{eff} with moduli dependence
- different from N-enhancement mechanisms: $f_{\text{eff}} \sim N^p f$ with p $\geq 1/2$,
  Dimopoulos-Kachru-McGreevy-Wacker (2005), Choi-Kim-Yung (2014), Bachlechner-Long-McAllister (2014/15), Junghans (2015)

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θ

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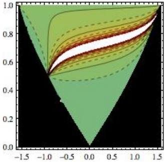

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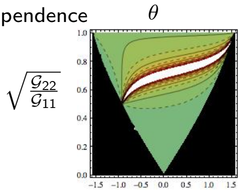

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Consistency & 4-Fermion Couplings

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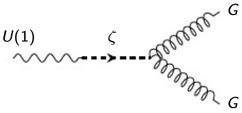
- $U(1)$ gauge invariance requires presence of chiral fermions ψ



- Integrating out massive $U(1)$ boson
  $\rightsquigarrow$ 1 axion ξ + 1 non-Abelian gauge group + chiral fermions

$$\mathcal{S} = \int \frac{1}{2} \mathrm{d}\xi \wedge \star_4 \mathrm{d}\xi - \frac{1}{8\pi^2} \frac{\xi}{f_\xi} \mathrm{Tr}(G \wedge G) - \frac{\mathcal{C}}{f_2^2} \underbrace{\mathcal{J}_\psi \wedge \star_4 \mathcal{J}_\psi}_{4-fermion} + \mathcal{L}_\psi$$

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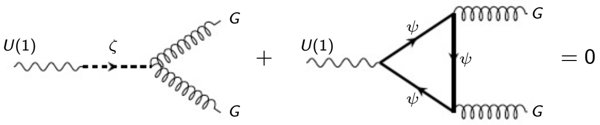

"reversed" GS mechanism

Aldazabel-Franco-Ibáñez-Rábadan-Uranga ('01)

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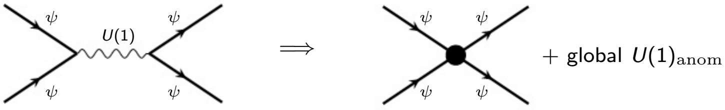

N-JL type couplings

see talk @ StringPheno 2016

$U(1)$ Breaking & Mass Generation

Shiu-W.S. (work in progress)

- at strong coupling for $SU(N)$ $\rightsquigarrow$ non-perturbative effects important
  - ⋆ Instantons ($\langle G \wedge G \rangle \neq 0$): $U(1) \to \mathbb{Z}_{n_f}$
    $\rightsquigarrow$ interactions between ψ and instantons 't Hooft (1976)

$$\mathcal{L}_{\text{'t Hooft}} = C\, e^{-\frac{8\pi^2}{g^2}+i\theta} \det(\overline{\psi}_L \psi_R) + h.c.$$

  - ⋆ Fermion condensate ($\langle \overline{\psi}_L \psi_R \rangle \neq 0$): $\mathbb{Z}_{n_f} \to \mathbb{Z}_2$ Casher (1979)
    $\rightsquigarrow$ fermion mass $M \sim -\frac{1}{M_{st}^2} \langle \overline{\psi}_L \psi_R \rangle$
- $E < \Lambda_s$: bound state $\overline{\psi}\psi$ $\to$ EFT for composite scalar $\Phi(x) = \sigma(x) e^{i\frac{\eta}{f}}$
- mass spectrum in vacuum

$f_\xi \ll f$	$f_\xi \sim f$	$f \ll f_\xi$
$\downarrow$	$\downarrow$	$\downarrow$
$m_\eta < m_\sigma \ll m_\xi$	$m_\xi, m_\eta < m_\sigma$	$m_\xi \ll m_\eta < m_\sigma$

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$$V = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2 + \Lambda_s^2 \left(M\Phi + M\Phi^\dagger + \kappa e^{i\frac{\xi}{f_\xi}} \det(\Phi) + \kappa e^{-i\frac{\xi}{f_\xi}} \det(\Phi^\dagger) \right)$$

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$$V = -\mu^2\sigma^2 + \frac{\lambda}{2}\sigma^4 + 2\Lambda_s^2 M\,\sigma \cos\frac{\eta}{f} + 2\kappa\Lambda_s^2\,\sigma \cos\left(\frac{\eta}{f} + \frac{\xi}{f_\xi} + \theta\right)$$

with vacuum $\langle\sigma\rangle = f \sim \Lambda_s$, $\langle\eta\rangle = 0 = \langle\xi\rangle$

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  - ★ Fermion condensate ($\langle \overline{\psi}_L \psi_R \rangle \neq 0$): $\mathbb{Z}_{n_f} \to \mathbb{Z}_2$ Casher (1979)
    $\rightsquigarrow$ fermion mass $M \sim -\frac{1}{M_{st}^2} \langle \overline{\psi}_L \psi_R \rangle$
- $E < \Lambda_s$: bound state $\overline{\psi}\psi$ $\to$ EFT for composite scalar $\Phi(x) = \sigma(x) e^{i\frac{\eta}{f}}$

$$V = -\mu^2\sigma^2 + \frac{\lambda}{2}\sigma^4 + 2\Lambda_s^2 M\, \sigma \cos\frac{\eta}{f} + 2\kappa\Lambda_s^2\, \sigma \cos\left(\frac{\eta}{f} + \frac{\xi}{f_\xi} + \theta\right)$$

with vacuum $\langle\sigma\rangle = f \sim \Lambda_s$, $\langle\eta\rangle = 0 = \langle\xi\rangle$

- mass spectrum in vacuum massive (σ, η) = INFLADRONS

$f_\xi \ll f$	$f_\xi \sim f$	$f \ll f_\xi$
$\downarrow$	$\downarrow$	$\downarrow$
$m_\eta < m_\sigma \ll m_\xi$	$m_\xi, m_\eta < m_\sigma$	$m_\xi \ll m_\eta < m_\sigma$

Phases of Axion Inflation

Shiu-W.S. (work in progress)

Witten, Veneziano ('79,'98)

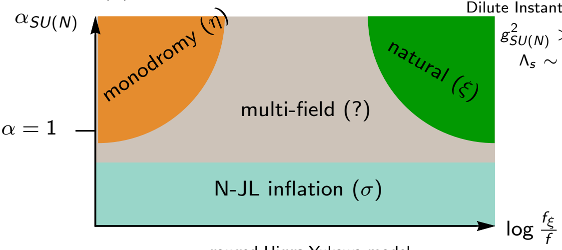
$SU(N)$ vacuum degeneracy

Dilute Instanton Gas

Honecker-Koltermann-W.S.

$g^2_{SU(N)} > 0.8$ @ $M_{\rm string}$

$\Lambda_s \sim f \sim 10^{16}$ GeV



$\sim$ gauged Higgs-Yukawa model

depends on UV boundary conditions

$\rightarrow$ chaotic inflation, α-attractor with $\alpha = 2$

Inagaki-Odintsov-Sakamoto ('15-'17)

see talk @ StringPheno 2016

Weak Gravity & Strong Dynamics

- WGC = criterium for field theory to be coupled to gravity
- electric WGC for $U(1)$: $\exists$ particle with $m \leq qM_{Pl}$
  - ⋆ In UV ($E > M_{st}$): 2 charged, massles axions ✓
  - ⋆ In IR ($E < M_{st}$): only global $U(1)$ ✓
- electric WGC for axions: $\exists$ instanton such that $S \lesssim \mathcal{O}(1)\frac{M_{Pl}}{f}$

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- electric WGC for axions: $\exists$ instanton such that $S \lesssim \mathcal{O}(1)\frac{M_{Pl}}{f}$
  - ⋆ In UV ($E > M_{st}$): (non dominant) D2-brane instantons ✓
  - ⋆ In IR ($E < \Lambda_s$): gauge instantons coupled to ξ appear to violate WGC ×

Conclusions and Outlook

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- Type II String compactifications $\rightsquigarrow$ rich 4dim EFT for axions
- At lower energy scales $\rightarrow$ N-JL interactions, instantons and fermion condensates generating dynamical masses
- $\exists$ a phase (parameter) space for inflationary models
- WGC fine in UV, violated in IR analogous to Saraswat (2016)

Open issues

- ☞ Verification of magnetic WGC see also Hebecker-Henkenjohann-Witkowski (2017), Dolan et al (2017)
- ☞ Detailed analysis for axion monodromy realisation Kaloper-Lawrence-Sorbo (2010)
- ☞ Full String Theory construction including moduli stabilisation

  Blumenhagen, Valenzuela, Zoccarato

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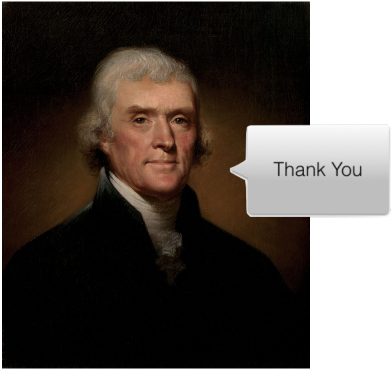
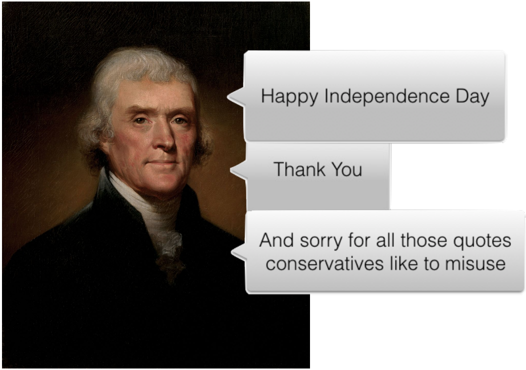

Thank You


Happy Independence Day
Thank You
And sorry for all those quotes conservatives like to misuse

Natural Inflation Phase

Inflaton = closed string axion

Viable inflationary model requires control over perturbative (& non-perturbative) quantum corrections:

- EFT for Infladrons Φ receives non-renormalisable corrections:
  - ⋆ derivative terms: $M_{UV}^{-4}\left|\partial\Phi^\dagger\partial\Phi\right|^2$, $M_{UV}^{-2}|\Phi|^2\left|\partial\Phi\right|^2$
  - ⋆ potential & mixed terms

  $\Rightarrow$ additionally suppressed by powers of $\frac{f}{f_\xi} \sim 10^{-3}$

- Loop-corrections to Φ
  - ⋆ 1-loop effective action $V^{1-loop} \sim (-\mu^2 + 3\lambda|\Phi|^2)^2 \left\{\ln\left(\frac{-\mu^2+3\lambda|\Phi|^2}{\Lambda_r^2}\right) - \frac{3}{2}\right\}$ Coleman-Weinberg ('73)

    $\rightsquigarrow$ proper resummation of eff. potential maintains vacuum structure
  - ⋆ Induced non-minimal coupling to gravity $\int\sqrt{-g}\,\xi|\Phi|^2 R$ Hill-Salopek ('92)

    Solving RGE for $\xi \rightarrow$ IR-fixed point $\xi = 0$
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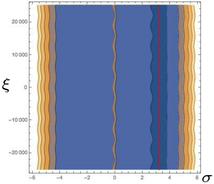
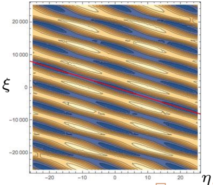




Stringy Axions and Stückelberg-mechanism

reviews: Baumann (2009), Baumann-McAllister (2009,2014), Westphal (2014), Grimm-Louis (2005), Grimm-Lopes (2011), Kerstan-Weigand (2011), Grimm-Louis (2004), Jockers-Louis (2005), Haack-Krefl-Lust-Van Proeyen-Zagermann (2006)

- Closed string axions a^i from dim. red. of p-forms $C_{(p)}$ on $\mathcal{M}_{1,3} \times \mathcal{X}_6/\Omega\mathcal{R}$ ($C_{(p)} \in$ RR-forms + NS 2-form in Type II)

$$a^i \equiv (2\pi)^{-1} \int_{\Sigma^i} C_{(p)}, \qquad p-\text{cycle } \Sigma^i \subset \mathcal{X}_6, \qquad i \in \{1, \ldots, \begin{matrix} h_{11} \\ h_{21}+1 \end{matrix} \}$$

  Kinetic terms for p-forms $C_{(p)} \in \rightsquigarrow$ kinetic terms for a^i

- Stack of N D-branes on $\mathcal{M}_{1,3} \times \Sigma^i \rightsquigarrow U(N)$ gauge theory in $4+p$ dim
  Reduction of D-brane CS-action $\rightsquigarrow$ couplings for a^i

  IIA
  - ⋆ $C_3 \wedge \text{Tr}(G \wedge G) \to$ anomal. coupling $a^i\, Tr(G \wedge G)$
  - ⋆ $C_5 \wedge F \to$ Stückelberg-coupling $(da^i - k^i A)^2$ under U(1)

- Subset of axions are eaten by anomalous $U(1)$'s $\rightsquigarrow$ survive as global symmetries
- Also possible in IIB for $C_{2,4}$ using D7-branes with magn. fluxes

2 Mixing Axions

- minimal set-up: 2 axions + 1 $U(1)$ + 1 Non-Abelian gauge group
- 1 axion eaten by $U(1)$ gauge boson, $\perp$ axion ξ with decay constant:

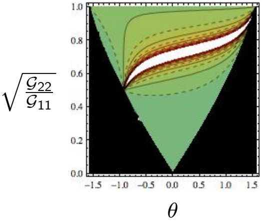
$$f_\xi = \frac{\sqrt{\lambda_+\lambda_-}M_{st}}{\cos\frac{\theta}{2}\left(\lambda_+ k^+ r_2 + \lambda_- k^- r_1\right) + \sin\frac{\theta}{2}\left(\lambda_- k^- r_2 - \lambda_+ k^+ r_1\right)}$$

with $\lambda_\pm$ eigenvalues of $\mathcal{G}_{ij}$ and $M_{st} \equiv \sqrt{\lambda_+(k^+)^2 + \lambda_-(k^-)^2}$

$$\cos\theta = \frac{\mathcal{G}_{11} - \mathcal{G}_{22}}{\lambda_+ - \lambda_-}, \quad \sin\theta = \frac{2\mathcal{G}_{12}}{\lambda_+ - \lambda_-}, \quad \begin{pmatrix} k^+ \\ k^- \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & -\cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} k^1 \\ k^2 \end{pmatrix}$$

- Contour plot representation of f_ξ (in units $\sqrt{\mathcal{G}_{11}}$)

$2k^1 = k^2 = 2r_1 = 2r_2$



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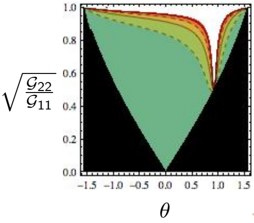
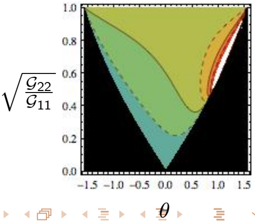


$k^1 = -2k^2 = r_1 = 2r_2$



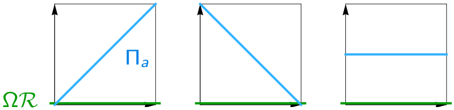
Axions & String Theory: Example

reviews: Blumenhagen-Cvetič-Langacker-Shiu ('05); Blumenhagen-Körs-Lüst-Stieberger ('06); Ibañez-Uranga ('12)

e.g. Type IIA D6-branes on $CY_3/\Omega\mathcal{R} \rightsquigarrow \Sigma^i = \Sigma^i_+ + \Sigma^i_-$

$$\int_{\Sigma^i_-} C_{(5)} \wedge F \neq 0 \qquad \rightsquigarrow \qquad \text{Stückelberg coupling for } a^i$$

$$T^6/\Omega\mathcal{R} \text{ with 4 } \Omega\mathcal{R}\text{-even 3-cycles } \underbrace{\Sigma_+^{i=0,1,2,3}}_{\text{4 axions } a^i} \text{ and 4 } \Omega\mathcal{R}\text{-odd 3-cycles } \Sigma_-^{i=0,1,2,3}$$



$$\Pi_a = \underbrace{\Sigma^0_+ - \Sigma^3_+}_{\Downarrow} + \underbrace{\Sigma^1_- - \Sigma^2_-}_{\Downarrow}$$

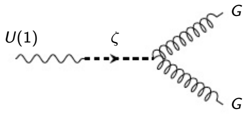
$$\begin{array}{cc} a^0 F_a \wedge F_a & (da^1 - A_a)^2 \\ -a^3 F_a \wedge F_a & (da^2 + A_a)^2 \end{array}$$

The road to NJL Models

Shiu-WS-Ye('15), Shiu-WS (work in progress)

- $U(1)$ gauge invariance: $A \to A + \mathrm{d}\chi$, $a \to a + k\chi$

$$\mathcal{S}_{sub} = \int \left[-\frac{M_{st}^2}{2} |\mathrm{d}a - kA|^2 - \frac{1}{4g_{U(1)}^2} |F|^2 - \underbrace{\frac{1}{8\pi^2} a \,\mathrm{Tr}(\, G \wedge G)}_{\text{not } U(1) \text{ invariant}} \right]$$



- @ intersection of two D-brane stacks with $U(N) \times U(1)$
  $\rightsquigarrow$ chiral matter in bifund. rep.

- EFT for generation-indep. $U(1)$ charges:

$$\mathcal{L} = -\frac{M_{st}^2}{2} |da - kA|^2 - \frac{1}{4g_{U(1)}^2} |F|^2 + i \sum_{i=1}^{N_f} \overline{\psi}_L^i \partial\!\!\!/ \psi_L^i + i \sum_{i=1}^{N_f} \overline{\psi}_R^i \partial\!\!\!/ \psi_R^i$$

$$+ \sum q_L \overline{\psi}_L^i A\!\!\!/ \psi_L^i + \sum q_R \overline{\psi}_R^i A\!\!\!/ \psi_R^i + \text{ coupling to } U(N)$$

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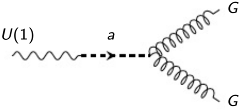
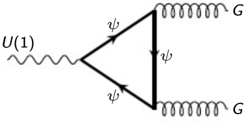
$+$ chiral fermions ψ



$+$



"reversed" GS mechanism

Antoniadis-Kiritsis-Rizos ('02)

Aldazabel-Franco-Ibáñez-Rábadan-Uranga ('01)

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$$(\Box, q_L) \quad \psi_L^i$$
$$(\Box, q_R) \quad \psi_R^i$$

with $q_L \neq q_R$

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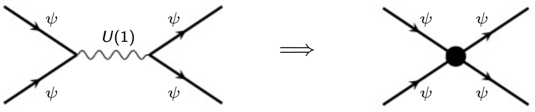
The road to NJL Models (II)

Shiu-WS-Ye('15), Shiu-WS(work in progress)

- Integrating out Stückelberg $U(1)$ through solving Lorenz gauge condition:

$$A \sim \frac{1}{M_{st}^2}\left(q_L \overline{\psi}^i_L \Gamma \psi^i_L + q_R \overline{\psi}^i_R \Gamma \psi^i_R\right) \qquad \rightsquigarrow \qquad \mathcal{L}_{4\psi} \sim \frac{q^2}{M_{st}^2}(\overline{\psi}\Gamma\psi)^2$$



$+$ global $U(1)_{\mathrm{anom}}$

- Using Fierz-identities $\rightsquigarrow$ NJL-type models with $N_f = 1$

$$\mathcal{L}_{4\psi} = \frac{q_L q_R}{2M_{st}^2}\left[(\overline{\psi}\psi)(\overline{\psi}\psi) - (\overline{\psi}\gamma^5\psi)(\overline{\psi}\gamma^5\psi)\right] + \ldots$$

NJL & Dynamical Mass Generation

Nambu-Jona-Lasinio ('61), review: Vogl-Weise ('91)

$\overline{\psi}\psi \to \overline{\psi}\psi\cos(2\alpha) + i\psi\gamma^5\psi\sin(2\alpha)$

$\overline{\psi}\gamma^5\psi \to \psi\gamma^5\psi\cos(2\alpha) + i\overline{\psi}\psi\sin(2\alpha)$

- NJL $= U(1)_{\rm chiral}$ invariant 4ψ-interactions $\quad \psi \to e^{i\alpha\gamma^5}\psi$

$$\mathcal{L}_{\rm NJL} = \overline{\psi} i \partial\!\!\!/ \psi + \frac{q_L q_R}{2M_{st}^2}\left[(\overline{\psi}\psi)(\overline{\psi}\psi) - (\overline{\psi}\gamma^5\psi)(\overline{\psi}\gamma^5\psi)\right]$$

- two phases:
  - ⋆ Wigner phase: $\langle\overline{\psi}\psi\rangle = 0 \rightsquigarrow U(1)_{\rm chiral}$ unbroken and ψ massless
  - ⋆ Nambu-Goldstone-phase : $\langle\overline{\psi}\psi\rangle \neq 0 \rightsquigarrow \exists\ m_\psi = -\frac{q_L q_R}{M_{st}^2}\langle\overline{\psi}\psi\rangle$ and $\not{U}(1)_{\rm chiral}$

  NG-phase requires satisfied self-consistency condition:

$$\text{GAP:} \quad m_\psi = \frac{4 i q_L q_R N}{M_{st}^2}\int \frac{d^4p}{(2\pi)^4}\frac{m_\psi}{p^2 - m_\psi^2}$$

  and $\exists\, m_\psi \neq 0$ at strong coupling: $\alpha_{U(1)}(\Lambda) > \frac{\pi}{N}$ for $\Lambda < M_{st}$

NJL & Dynamical Mass Generation (II)

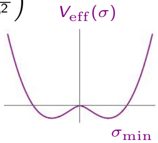
- Verify minimum in NG-phase in large N limit Gross-Neveu ('74)
  - ⋆ auxiliary fields σ, π: $\begin{pmatrix} \sigma \\ \pi \end{pmatrix} \to \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$

$$\mathcal{L}_\sigma = \overline{\psi} i \partial\!\!\!/ \psi - \frac{1}{2g^2}(\sigma^2 + \pi^2) + \left(\sigma \overline{\psi}\psi + i\pi\overline{\psi}\gamma^5\psi\right), \qquad g^2 = \frac{q_L q_R}{M_{st}^2}$$

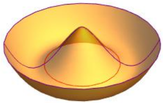
  - ⋆ effective potential $V_{\rm eff}(\sigma) = \frac{1}{2g^2}\sigma^2 - 2N \int \frac{d^4p}{(2\pi)^4} \ln\left(1 + \frac{\sigma^2}{p^2}\right)$

    $\rightsquigarrow$ minimum: $\frac{dV_{\rm eff}}{d\sigma}\Big|_{\sigma=\sigma_{\rm min}} = 0 \to$ GAP eq

$$V_{\rm eff}(\sigma_{\rm min}) < 0$$



- Bound (scalar) states: (Salpeter-Bethe ('51) or poles of $G^{(4)}_{4\psi}$)
  - ⋆ 0^+ state $\sigma = g\overline{\psi}\psi \quad \rightsquigarrow m^2_{0^+} = 4m^2_\psi$
  - ⋆ 0^- state $\pi = g\overline{\psi}i\gamma^5\psi \rightsquigarrow m^2_{0^-} = 0$



First Inflationary Steps

- Gradient expansion $\rightsquigarrow$ EFT for (σ, π) with inflaton$= \sigma$

$$\mathcal{L}_{(\sigma,\pi)} = \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}(\partial\pi)^2 - \frac{1}{2}4m_\psi^2\sigma^2 - \frac{\lambda}{4}\sigma^4 + \dots$$

- RGE-analysis with conformal coupling to gravity $\rightsquigarrow$ R^2-type inflation
  Hill-Salopek ('92), Inagaki-Odintsov-Sakamoto ('15)

$$V_{(\sigma,\pi)}^{\mathrm{RGE}} = \frac{1}{1 + \frac{D}{6}(\sigma^2)^{1/(1+\frac{\alpha}{\alpha_c})}} \left(\frac{B}{2}\sigma^2 + \frac{C}{4}(\sigma^4)^{1/(1+\frac{\alpha}{\alpha_c})} \right)$$

$\rightsquigarrow$ compatible with Planck 2015 data:
$n_s = 0.961$, $r = 0.0083$ @ weak coupling $\alpha \ll 1$

- Strong $SU(N)$ dynamics can spoil (σ, π) set-up: 't Hooft (1976)
  - ⋆ $SU(N)$ gauge instantons produce $\not{U}(1)$ coupling $\kappa\, \mathrm{det}(\overline{\psi}(1+\gamma^5)\psi) + h.c.$
  - ⋆ $m_\pi^2 \sim |\kappa|^2 < m_\sigma^2 = 4m_\psi^2 + m_\pi^2 \rightsquigarrow \pi =$ inflaton ?
- $\exists$ remaining string axions coupling to $SU(N)$ can play rôle of inflaton?

First Inflationary Steps