Axion Inflation and the
Lattice Weak Gravity Conjecture

Tom Rudelius
Harvard/IAS
Based On

• 1409.5793/hep-th, 1503.00795/hep-th
• 1506.03447/hep-th, 1509.06374/hep-th
  • with Ben Heidenreich, Matthew Reece
• To appear
  • with Ben Heidenreich, Cody Long, Liam McAllister, Matthew Reece, John Stout
Outline

I. The Weak Gravity Conjecture
II. Axion Inflation
III. The WGC and Inflation
   1. Mild WGC and Inflation
   2. Lattice WGC and Inflation
The Weak Gravity Conjecture
The Weak Gravity Conjecture

The WGC: In any $U(1)$ gauge theory that admits a UV completion with gravity, there must exist a state of charge $q$, mass $m$, such that

$$\frac{q}{m} \geq \frac{Q}{M} \bigg|_{\text{extremal}}$$

Arkani-Hamed, Motl, Nicolis, Vafa, ’06

(see talks by Gary Shiu, Ben Heidenreich)
Why should the WGC be true?

- If not, extremal BH will be unable to decay.
- If not, near-extremal BH will move towards extremality.
Why should the WGC be true?

• Many examples from string theory, KK theory:

\[ \text{Q} = \text{M} \]
The WGC for Multiple $U(1)$s

1. Take theory with $N$ 1-form $U(1)$s.

2. Consider charge-to-mass vectors $\vec{z}_i = \frac{\vec{q}_i}{m_i} M_p$.

3. WGC: Convex Hull of $\{\vec{z}_i\}$ must contain unit ball in $\mathbb{R}^N$.

Cheung, Remmen ’14
The sLWGC

Sublattice Weak Gravity Conjecture: There exists some sublattice of the full charge lattice with particles satisfying $|\vec{q}|/m \geq |\vec{Q}|/M_{\text{ext}}$. 
Axion Inflation
Inflation

Problem: Why is the universe so flat and so homogeneous?

Solution: Inflation—postulated period of exponential expansion in the early universe
Axion Inflation

- Axions (scalars with perturbative shift symmetry) acquire periodic potential via instanton effects:

\[ V(\phi) = \Lambda^4 (1 - \cos \frac{\phi}{f}) + \ldots \]

“axion decay constant”

- Need \( f > M_p \) for inflation
The WGC and Inflation
The Generalized WGC

The generalized WGC: In any $p$-form theory in $d$ dimensions, there must exist an electrically charged object of dimension $p - 1$ and a magnetically charged object of dimension $d - p - 1$ with

\[ T_{el} \lesssim \left( \frac{g^2}{G_N} \right)^{1/2}, \quad T_{mag} \lesssim \left( \frac{1}{g^2 G_N} \right)^{1/2} \]
WGC and Axion Inflation

Consider 0-form $\phi$ with decay constant $f$. Charged object is an instanton of action $S$.

$$WGC \Rightarrow 1/(fS) \gtrsim 1/M_p.$$  
$$S > 1 + WGC \Rightarrow f \lesssim M_p$$

Incompatible with inflation!

Agrees with string theory

Banks, Dine, Fox, Gorbatov ’03
Multi-Axion Inflation

- Recruit additional axions:

\[ f_{\text{eff}} \sim |\cot \theta| M_p \]

Decay Constant Alignment

Kim, Nilles, Peloso ’04

\[ f_{\text{eff}} \sim \sqrt{N} M_p \]

N-flation

Dimopoulos, Kachru, McGreevy, Wacker ’05
Multi-Axion Inflation

- Simplest models violate convex hull condition

Decay Constant Alignment

$N$-flation
Multi-Axion Inflation

• Field space diameters bounded!

\[ f_{eff} \sim M_p \]

TR ’14, ’15
Brown, Cottrell, Shiu, Soler ’15
Montero, Uranga, Valenzuela ’15

Decay Constant Alignment

\[ f_{eff} \sim \sqrt{N} f \sim M_p \]

\[ f \sim M_p / \sqrt{N} \]

N-flation
(s)LWGC and Inflation

- Model with $P$ instantons, $N$ axions, WGC bounds assume

\[ P = N \]

- (s)LWGC says

\[ P = \infty \]

- Need to revisit WGC constraints in light of new conjecture
Two Setups

- Consider square lattice $\Lambda$ of instantons saturating WGC bound:

$$S = |\vec{Q}| := \sqrt{\vec{Q} \cdot \vec{Q}}$$

1) $$S = |\vec{Q}| := \sqrt{\vec{Q} \cdot \vec{Q}}$$

2) $$S = |\vec{Q}| := \sum_i |Q_i| t^i$$
First Setup

- Consider square lattice $\Lambda$ of instantons saturating WGC bound:

\[
\mathcal{L} \supset -\frac{1}{2} K_{a b} \partial \phi^a \partial \phi^b - \sum_{\vec{Q} \in \Lambda} e^{-|\vec{Q}|} \cos(\vec{Q} \cdot \vec{\phi} + \delta_{\vec{Q}}) \\
|\vec{Q}| := \sqrt{\vec{Q} \cdot K^{-1} \cdot \vec{Q}}
\]
A Loophole

• Set $\delta \vec{Q} = \delta = \text{const.}, N = 1,$

$$V(\phi) = \sum_{n \in \mathbb{Z}} e^{-n/f_0} \cos \frac{n\phi}{f_0} + \delta$$

$$= V_1 + V_0 \frac{\cos(\phi/f_0 + \delta) - e^{1/f_0} \cos \delta}{\cosh 1/f_0 - \cos \phi/f_0}.$$
A Second Loophole

- Scrunch lattice along one diagonal:

- Effect on potential is subleading in $N^{-1}$ in scrunched direction, find

$$f_{\text{eff}} \sim \sqrt{N} M_p$$
A Third Loophole

• Set $N = 2$, take two aligned instantons to have $S_{\bar{Q}} \ll |\vec{Q}|$ and dominate potential

• Gives rise to standard KNP alignment

Kim, Nilles, Peloso ’04
Generic Constraints

• More generally, take random phases $\delta \vec{Q}$
• Given direction $\vec{e}_0$ in charge lattice, decompose potential into harmonics
Phasor Notation

• Introduce phasor notation,

\[ \mathcal{L} \supset -\frac{1}{2}K_{ab}\partial\phi^a\partial\phi^b - \sum_{\vec{Q} \in \Lambda} Z_{\vec{Q}} e^{i\vec{Q} \cdot \vec{\phi}} \]

\[ Z_{\vec{Q}} = e^{-|\vec{Q}| + i\delta_{\vec{Q}}} \]

• Total contribution to nth harmonic is then

\[ Z_n = \sum_{\vec{Q}, \sum_i Q_i = |n|} Z_{\vec{Q}} \]
Gaussian Approximation

- For large number of instantons, can use CLT to write:

\[ Z_n \sim \mathcal{N}(0, \sigma_n^2) \]

\[ \sigma_n^2 = \sum e^{-2|\vec{Q}|} \]

\[ \vec{Q}, \sum_i Q_i = |n| \]

Higher harmonics suppressed \( \Leftrightarrow \sigma_n^2 \ll \sigma_1^2 \)
Estimating $\sigma_n^2$

- Want to estimate sum

$$\sigma_n^2 = \sum e^{-2|\tilde{Q}|}$$

$\tilde{Q}, \sum_i Q_i = |n|$

- Method 1: use instantons of small charges and ignore the rest (valid for small decay constants)

- Method 2: use Poisson resummation (valid for large decay constants)

  c.f. talks by Liam McAllister, John Stout
Volume Bound

• For general $K_{ab}$, define

$$\text{vol } \Phi = \frac{M_p^N (2\pi)^N}{|\Lambda|}$$

with $|\Lambda|$ the volume of the unit cell, $\Phi$ the fundamental domain of axion moduli space:

Decay Constant Alignment

$N$-flation
Volume Bound (cont.)

- Demanding $\sigma_n^2 \ll 1$ and approximating, get

$$\log[\text{vol } \Phi] \preceq \log[\text{vol } D_N(2M_p)] + O(\log N)$$

with $D_N(R)$ the $N$-ball of radius $R$. 
Volume Bound and $N$-flation

- Isotropic $N$-flation violates the volume bound

- But, decay constant alignment survives
Volume Bound and Alignment

- Set $K_{ab} \in \text{Wish}_N(\sigma^2, N)$
  \[ \Rightarrow f_{\text{eff}} \lesssim O(1M_p) \]

- Set $K_{ab}^{-1} \in \text{Wish}_N(\sigma^2, N)$
  \[ \Rightarrow f_{\text{eff}} \lesssim O(NM_p) \]
Second Setup

• Square lattice $\Lambda$ of instantons saturating WGC bound, now with linear action sum:

\[
\mathcal{L} \supset -\frac{1}{2} K_{ab} \partial \phi^a \partial \phi^b - \sum_{\vec{Q} \in \Lambda} e^{-|\vec{Q}|} \cos(\vec{Q} \cdot \vec{\phi} + \delta \vec{Q})
\]

\[
|\vec{Q}| := \sum_i |Q_i| t^i
\]
Convex Hull Condition

- Expect CHC satisfied by charge 1 instantons (c.f. “mild WGC” constraints):

- Would need higher harmonics suppressed to get $N$-flation
\(N\)-flation

- Higher harmonics actually become \textit{worse} as \(N\) increases:

\[
\frac{\sigma_2^2}{\sigma_1^2}
\]

- No \(N\)-flation!
Summary

• The WGC is incompatible with some models of axion inflation

• If the (s)LWGC is true:
  • Isotropic N-flation would be ruled out
  • Some decay constant alignment models would be ruled out
  • But, some would be allowed

• See talks by Liam McAllister, John Stout for further details
Open Questions

• Can one close the aforementioned loopholes?
• If not, can one exploit one of these opportunities in a stringy model of inflation?
• What more can the WGC tell us about low-energy physics? (see talks by Valenzuela, Blumenhagen, Hebecker, Montero, and Ooguri)