Exotic Representations and Singular 7-branes in F-theory

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String Pheno
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Based on 1706.08194 with D. Klevers, D. Morrison, and W. Taylor

Overarching Question

Which charged matter representations can be obtained in F-theory?

Certain reps. obtained fairly easily in F-theory

- Fundamental, 2-index antisymmetric, and adjoint of SU(N)
- $q = 1, 2$ for $U(1)$

Others are more exotic, tougher to get

- Symmetric (□□) of SU(N)
- 3-index symmetric (□□□) of SU(2)
- $q \geq 3$ for $U(1)$

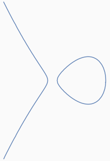
Goal: Better understand how to explicitly construct models with exotic representations

Higher Genus Representations

- Certain reps. involve 7-branes wrapped on higher genus divisors
- Exotic reps. often localized at singular points

For SU(N)

Smooth Curve with Genus g

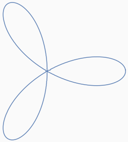


g Adjoints

Double Point Singularity

Adjoint or $\square\square + \begin{smallmatrix}\square\\ \square\end{smallmatrix}$

Triple Point Singularity



3 Adjoints or $\square\square\square + 2 \times \square$

Difficulties Obtaining Exotic Higher Genus Reps

When do you get adjoints vs. exotics?

- [Sadov '96]: Double points give □□
- [Morrison, Taylor '11]: Double points can give either adjoints or □□
- How can we distinguish two situations?

Previous models found indirectly

- Unhiggs $U(1)\times U(1)\rightarrow SU(3)$ w/ □□ [Cvetic, Klevers, Piragua, Taylor '15]
- Higgs $SU(6)\rightarrow SU(3)$ w/ □□ [Anderson, Gray, NR, Taylor '15]
- Unhiggs $U(1)\rightarrow SU(2)$ w/ □□□ [Klevers, Taylor '16]
- Want a more direct way of constructing

Weierstrass models w/ exotic higher-genus reps. have non-Tate structure

- How can we derive and explain this structure?

An Example of Non-Tate Structure

$$y^2 = x^3 + fx + g \qquad \Delta = 4f^3 + 27g^2 \qquad \text{SU(N): } \Delta \propto \sigma^N$$

Expand f and g as

$$f = f_0 + f_1\sigma + f_2\sigma^2 + \dots \qquad g = g_0 + g_1\sigma + g_2\sigma^2 + \dots$$

For zeroth order cancellation: $4{f_0}^3 + 27g_0^2 \equiv 0 \bmod \sigma$

Option 1: *Exact Cancellation (Tate's algorithm)*

$$f_0 = -3\phi^2 \qquad g_0 = 2\phi^3.$$

$$4{f_0}^3 + 27{g_0}^2 = 0$$

Option 2: *Suppose $\sigma = \xi^3 - b\eta^3$ w/ triple point at $\xi = \eta = 0$*

$$f_0 = -3b\xi\eta \qquad g_0 = 2b^2\eta^3$$

$$4f_0^3 + 27{g_0}^2 = -108b^3\eta^3\left(\xi^3 - b\eta^3\right)$$

Models with exotics have structures similar to Option 2

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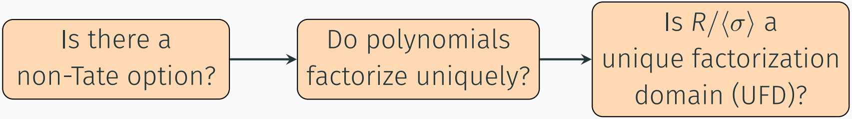
Singular Curves and Non-UFD Structure.

Why is non-Tate structure allowed?

- Consider quotient ring $R/\langle\sigma\rangle$ $(x = x + a\sigma)$
- Cancellation condition becomes

$$4{f_0}^3 = -27g_0^2$$



- **When σ is singular, quotient ring is not a UFD.**
  - One can consider normalization of $\sigma = 0$
  - Add elements from field of fractions to $R/\langle\sigma\rangle$
  - Resulting ring is a UFD **(normalized intrinsic ring)**

Tuning with the Normalized Intrinsic Ring

For the curve $\xi^3 - b\eta^3$.

1. Introduce new parameter $\tilde{B}$, with

$$\tilde{B}^3 = b \qquad\qquad \xi = \tilde{B}\eta$$

   After adding $\tilde{B}$, ring is a UFD

2. Start with the generic tunings

$$f_0 \sim -3\phi^2 \qquad\qquad g_0 \sim 2\phi^3$$

3. Let ϕ depend on $\tilde{B}$, but f_0, g_0 cannot directly depend on $\tilde{B}$

$$\phi = \tilde{B}^2\eta$$

$$f_0 \sim -3\tilde{B}^4\eta^2 \rightarrow -3b\xi\eta \qquad\qquad g_0 \sim 2\tilde{B}^6\eta^3 \rightarrow 2b^2\xi^3$$

These are the non-Tate tunings from before.

Models that can be derived

SU(2) with triple points and □□□ matter

- Slghtly more general model than [Klevers, Taylor ’16].

SU(N) with double points and □□ matter

- Split condition implemented using normalized instrinsic ring
  - Determines whether double points support adjoints or symmetrics

Reps. that cannot be realized

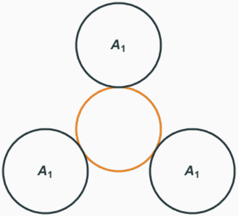
Can we realize □□□□ of SU(2), □□□ of SU(3), ...?

- Seemingly consistent SUGRA models that have these reps.
- Attempts to tune them lead to codim. 2 (4,6) singularities

Assertion: These reps cannot be realized in 6D F-theory models

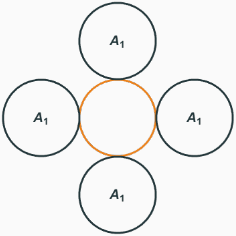
- These reps involve enhancements to affine Dynkin diagrams
- Need to shrink extra node

□□□ of SU(2)



$A_1{}^3 \to D_4$

□□□□ of SU(2)



$A_1{}^4 \to \hat{D}_4$

U(1) Charges in F-theory

In F-theory, U(1)'s $\leftrightarrow$ rational sections of the elliptic fibration

- Well-known models with $q = 1, 2$ [Morrison, Park '11]
- There is a $q = 3$ model
  - [Klevers, Mayorga Pena, Piragua, Oehlmann, Reuter '14]
  - Found within group of toric constructions.
  - Different structure from Morrison-Park form
- $q \geq 4$ remains challenging

Q: How can we understand and derive models with $q \geq 3$?

- Can be found by Higgsing models with higher-genus exotics.
- Non-UFD structures should be present in Weierstrass model

Rational Sections

For a single U(1), need an additional rational section $[\hat{x} : \hat{y} : \hat{z}]$

$$\text{Global Weierstrass Form: } \hat{y}^2 - \hat{x}^3 = \hat{z}^4 \left(f\hat{x} + g\hat{z}^2 \right)$$

LHS has similar algebraic form to discriminant.

Strategy for tuning models:

1. Expand $\hat{x}$, $\hat{y}$ as series in $\hat{z}$
2. Tune $\hat{x}$ and $\hat{y}$ so that $\hat{y}^2 - \hat{x}^3 \propto \hat{z}^4$
    - Similar to tuning an I_4 singularity
3. If necessary, further tune $\hat{x}$ and $\hat{y}$ so that $\hat{y}^2 - \hat{x}^3$ takes form above
4. Read off f and g

Obtaining U(1)s with q ≥ 3

Using UFD tunings leads to Morrison-Park form

- Much like UFD tuning of I_4^{ns} singularities

Suppose $\hat{z}$ has singular structure

- $R/\langle \hat{z} \rangle$ may not be a UFD
- Now can have non-UFD structure in the tunings
- Use normalized intrinsic ring techniques to tune U(1)

Example: Charge 3

- Perform tuning using $\hat{z} = b_2\eta_a^2 + 2b_1\eta_a\eta_b + b_0\eta_b^2$
  - Singular at $\eta_a = \eta_b = 0$
- Leads to generalization of previous $q = 3$ model

Conclusions and Future Work

- Using normalized intrinsic ring techniques, one can systemically construct Weierstrass model with higher-genus matter
- Many reps (such as □□□□ of SU(2), □□□ of SU(3)) may not be realizable in F-theory
  - Interesting candidates for F-theory swampland
  - Suggests F-theory can only give a few exotic reps. beyond easily constructed ones
- Normalized intrinsic ring may help construct U(1) models with $q \geq 3$

Future Work

- Better understand non-realizable representations
- Compare to other string constructions
  - In heterotic: [Lewellen '89], [Dienes, March-Russell '96]
- Can $q \geq 4$ models be constructed?

Backup Slides

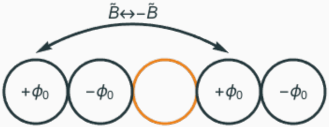
Symmetrics and the Split Condition

To tune SU(N) on $\sigma = \xi^2 - b\eta^2$:

1. Introduce parameter $\tilde{B}$: $\tilde{B}^2 = b$, $\tilde{B}\eta = \xi$
2. Tunings: $f = -3\phi^2 + \ldots$ $g = 2\phi^2 + \ldots$
3. Must implement Split Condition: $\phi = \phi_0^2$
4. Near double point, curve looks like $(\xi + \tilde{B}\eta)(\xi - \tilde{B}\eta)$
    - The two "components" should be identified with each other

Adjoint

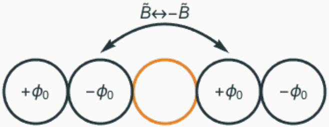
Generic ϕ_0



Dynkin index $[1, 1]$

Symmetric

$\phi_0 = \tilde{B}$



Dynkin index $[2, 0]$

Obtaining Morrison-Park form

1. Write $\hat{x}$ and $\hat{y}$ as

$$\hat{x} = x_0 + x_1\hat{z} + x_2\hat{z}^2 + \dots \qquad \hat{y} = y_0 + y_1\hat{z} + y_2\hat{z}^2 \dots$$

2. To have $\hat{y}^2 - \hat{x}^3 \propto \hat{z}^4$, use UFD I_4 tuning with altered coefficients:

$$\hat{x} = \phi^2 + x_2\,\hat{z}^2 \qquad \hat{y} = \phi^3 + \frac{3}{2}\phi\,x_2\,\hat{z}^2 + y_4\,\hat{z}^4$$

3. Without any further tuning,

$$\hat{y}^2 - \hat{x}^3 = \hat{z}^4\left[\underbrace{\left(\frac{-3}{4}x_2^2 + 2\phi y_4 + f_2\hat{z}^2\right)}_{f}\hat{x} + \underbrace{\left(-\frac{1}{4}x_2^3 + x_2 y_4\phi + y_4\hat{z}^2 - f_2\hat{x}\right)}_{g}\hat{z}^2\right]$$

   Can read off f and g from this

4. With replacements

$$\hat{z} \to b \qquad x_2 \to -\frac{2}{3}c_2 \qquad \phi \to c_3 \qquad y_4 \to \frac{1}{2}c_1 \qquad f_2 \to -c_0$$

   recover Morrison-Park form