Moduli and Maps in Type IIB

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Based on work with James Gray arXiv:1707.XXXXX

- In the presence of fluxes complicated geometries (not necessarily CY) arise in string theory solutions.

- The moduli space of some Heterotic string compactifications can be written as a kernel of maps between Dolbeault cohomology groups. In particular explicit examples they can be computed.

- We want to see how much of these techniques can be applied to Type IIB string theory in the presence of fluxes.

Moduli of Heterotic string in the presence of fluxes

- General condition for SUSY vacua in Heterotic string has been found by:

  -Strominger, Superstrings with Torsion, Nuclear Physics B274 (1986)

  -C. M. Hull, Compactification of the Heterotic Superstring, Physics Letters B (1986)

  -G. L. Cardoso, G. Curio, G. Dall'Agata, D. Lust, NonKahler string backgrounds and their five torsion classes, arXiv: 0211118

- An analysis of the moduli space of Strominger system, has been carried out by:

  -Lara Anderson, James Gray and Eric Sharpe arXiv: 1402.1532

  -Xenia de la Ossa and Eirik E. Svanes arXiv: 1402.1725

SUSY Background For Type IIB

- Condition on the SUSY vacua in Type II string theory for the backgrounds with SU(3) structure has been found by:

  - M. Grana, R. Minasian, M. Petrini and A. Tomasiello, arXiv: 0406137v2

- In Type IIB there are three types of vacua (A, B and C) and an interpolating one.

- We have shown that a very similar analysis to the Heterotic case, can be done for these three cases.

  -See L. Martucci arXiv: 0902.4031 for an example of recent related work

Type B vacua:

Internal space is a conformal CY

Includes:

$$e^{\phi} F_3 = i(H_{(2,1)} - H_{(1,2)}) \qquad F_3 = dC_2 - C_0 H$$

$$H_{(0,3)} = F_{(0,3)} = 0 \qquad d\tau = 0 \qquad \tau = C_0 + i e^{-\phi}$$

H and F are primitive

$$H_{(0,3)} = 0 \quad \text{or} \quad \Pi_i^{(+)p} \Pi_j^{(+)q} \Pi_k^{(+)r} H_{pqr} = 0 \quad , \quad \Pi_i^{(+)j} = \frac{1}{2}(\mathbb{I} + iJ)_i^j$$

Fluctuation ⟹

$$-\tfrac{3i}{2} \delta J^d_{[\bar{a}} (H)_{\bar{b}\bar{c}]d} = (\delta H)_{\bar{a}\bar{b}\bar{c}} = \bar{\partial}(\delta B)_{\bar{a}\bar{b}\bar{c}}$$

Consider the map $F : H^1(TX) \to H^3(X)$

$$\delta J^d_{\bar{a}} \mapsto -\frac{3i}{2} \delta J^d_{[\bar{a}} (H)_{\bar{b}\bar{c}]d}$$

This gives a constraint on the complex structure moduli: $\delta J \in \mathrm{Ker}(F)$

$$e^{\phi} F = i(H_{(2,1)} - H_{(1,2)})$$

Fluctuation:

$$\delta\bar{\tau} H_{\bar{a}\bar{b}c} + 2e^{-\phi}\delta J^{d}_{[\bar{a}} H_{\bar{b}]cd} = d(\delta C_2 - \bar{\tau}\delta B)_{\bar{a}\bar{b}c}$$

$$\delta\bar{\tau} H_{\bar{a}\bar{b}c} + 2e^{-\phi}\delta J^{d}_{[\bar{a}} H_{\bar{b}]cd} = 2\bar{\partial}_{[\bar{a}}(\delta C_2 - \bar{\tau}\delta B + \frac{1}{2}\Lambda)_{\bar{b}]c}$$

Consider the map

$$G : H^0(X) \oplus H^1(TX) \to H^2(TX^{\vee})$$

$$(\delta\bar{\tau}, \delta J) \mapsto \delta\bar{\tau} H_{\bar{a}\bar{b}c} + 2e^{-\phi}\delta J^{d}_{[\bar{a}} H_{\bar{b}]cd}$$

This gives another constraint on the moduli of complex structure: $(\delta\bar{\tau}, \delta J) \in \mathrm{Ker}(G)$

Computing the moduli in an explicit example

$$H_{(1,2)} \in H^2(TX^\vee)$$

$$H^1(TX) \to H^3(X)$$

$$H^0(X) \oplus H^1(TX) \to H^2(TX^\vee)$$

$$H_{(2,1)} \in H^1(\textstyle\bigwedge^2 TX^\vee)$$

$$H_{(1,2)} \in H^2(TX^\vee)$$

$$X = [\mathbb{P}^4|5]/\mathbb{Z}_5 \times \mathbb{Z}_5$$

$$p = x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5 = 0$$

$$H^1(TX) \simeq H^2(TX^\vee) \simeq H^1(\textstyle\bigwedge^2 TX^\vee)$$ With dimension 5

$$H^0(X) \simeq H^3(X) \simeq \mathbb{C}$$ With dimension 1

It shows it is possible to stabilize all the complex structure and axion dilaton moduli using these two maps

In polynomial representation of these spaces,
$H^1(TX)$ is a subspace of degree 5 homogeneous polynomials and
$H^2(TX^\vee)$ is a subspace of inverse degree 5 homogeneous polynomials and
$H^1(\bigwedge^2 TX^\vee)$ is a subspace of inverse degree 10 homogeneous polynomials.

e.g. Choice of flux with $\delta\tau = 0$

$$H_{(2,1)} = \frac{1}{x_0^3 x_1^2 x_2^2 x_3^2} + \frac{1}{x_0^2 x_1^3 x_2^3 x_3^2} + \frac{1}{x_0^2 x_1^2 x_2^3 x_4^3} + \frac{1}{x_0^2 x_2^2 x_3^3 x_4^3} + \frac{1}{x_0^3 x_1^2 x_3^2 x_4^3} + \frac{1}{x_1^3 x_2^2 x_3^2 x_4^3} + \frac{1}{x_0^3 x_1^3 x_2^2 x_4^2} + \frac{1}{x_0^2 x_1^3 x_3^3 x_4^2} + \frac{1}{x_1^2 x_2^3 x_3^3 x_4^2} + \frac{1}{x_0^3 x_2^3 x_3^2 x_4^2}$$

allowed complex structure moduli: $a(x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5) + b x_0 x_1 x_2 x_0 x_3 x_4$

Difference to Heterotic calculation

Subtlety

It is not easy to find the complex conjugate of the choice of flux in polynomial language in general but we can do it if we know the Kahler potential of complex structure moduli.

Simplification c.f. Heterotic

There is always a primitive representative for “H” in a given cohomology class, In contrast to the Heterotic case where it is difficult to satisfy $g^{\bar{a}b} F_{\bar{a}b} = 0$ in practice.

Conclusion

- There has been developed methods for computing the moduli space of the Strominger system in Heterotic string theory, which we have tried to apply to type IIB.

- In an explicit computation we have shown that there are improvements and difficulties in comparison to Heterotic case.

THANKS FOR YOUR ATTENTION