\[ V(\phi) = \Lambda^4 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right] \]

\[ m^2 = \frac{\Lambda^4}{f^2} \]

\[ |\phi| \lesssim \pi f \]

\[ \mathcal{L} \supset \frac{\phi}{f} F \tilde{F}, \frac{\partial_{\mu} \phi}{f} \bar{\psi} \gamma^{\mu} \gamma^5 \psi \]

\[ g \equiv \frac{\alpha}{2\pi} \frac{1}{f} \mathcal{N} \]
CURRENT LIMITS

ALPS

RUNNING EXPERIMENTS:
- IAXO
- MADMAX
- ADMX
- ETC...

Areas being probed
\[ \ddot{\varphi} + 3h \dot{\varphi} + \varphi + \kappa^2 \sin(\varphi) = 0 \]

sensitive to initial conditions

\[ \frac{\Delta \varphi_{\text{initial}}}{2\pi} = 0.1 \]

vast difference in EOS
\[ \ddot{\varphi} + 3h \dot{\varphi} + \varphi + \kappa^2 \sin(\varphi) = 0 \]

sensitive to initial conditions

\[ \frac{\Delta \varphi_{\text{initial}}}{2\pi} = 0.1 \]

vast difference in EOS
FLUCTUATION GROWTH

stability/instability bands
FLUCTUATION GROWTH

\[ \kappa = 20, \quad \frac{\varphi_{\text{initial}}}{2\pi} = 100 \]
OVERABUNDANCE IN ALP CDM MODELS

CRISIS AT SMALL SCALES

- missing satellites problem - cdm predicts more

- "banker’s delight" problem - cdm predicts too much mass

- cusp vs core problem - cdm predicts nfw cusps not cores
FLUCTUATIONS AS WARM COMPONENTS

MONODROMY DARK MATTER 2.0

\[ \varphi(\tau, \nu) = \varphi_0(\tau) + \delta\varphi(\tau, \nu) \]

Hill type equation:

\[ \ddot{c}_k + p_k(\tau) c_k = 0 \]

\[ p_k(\tau) = 1 + k^2 + \kappa^2 \cos \varphi_0(\tau) \]
**FLUCTUATIONS AS WARM COMPONENTS**

**HILL’S DETERMINANTAL METHOD**

\[
\cos(\varphi_0(z/\Omega)) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(2nz)
\]

\[
c_k'' + \left[ \theta_0 + \sum_{n=1}^{N} 2\theta_n \cos(2nz) \right] c_k = 0
\]

\[
\theta_0 = \Omega^{-2} \left( 1 + k^2 + \kappa^2 \frac{a_0}{2} \right)
\]

\[
\theta_n = \Omega^{-2} \kappa^2 \frac{a_n}{2}
\]

series truncates

Floquet theory:

\[
e^{\mu z} F(z) = e^{\mu \tau} \sum_{m=-\infty}^{\infty} b_m e^{2imz}
\]

\[
D_{mm} b_m = 0
\]

\[
D_{mn}(i\mu) = \frac{(i\mu - 2m)^2}{4m^2 - \theta_0} \delta_{mn} - \frac{\theta_{m-n}}{4m^2 - \theta_0}
\]

Hill’s determinant:

\[
\Delta(i\mu) \equiv |D_{mn}(i\mu)|
\]
HILL’S DETERMINANTAL METHOD

\[
\sin^2\left(\frac{\pi}{2} i \mu\right) = \Delta(0) \sin^2\left(\frac{\pi}{2} \sqrt{\theta_0}\right)
\]

\[
\Delta(0) = \begin{pmatrix}
  \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
  \cdots & 1 & -\theta_1 & -\theta_2 & -\theta_3 & -\theta_4 \\
  \cdots & -\frac{1}{4-\theta_0} & 1 & -\theta_1 & -\theta_2 & -\theta_3 \\
  \cdots & 0 & -\frac{1}{4-\theta_0} & 1 & -\theta_1 & -\theta_2 \\
  \cdots & -\frac{1}{4-\theta_0} & 0 & -\frac{1}{4-\theta_0} & 1 & -\theta_1 \\
  \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{pmatrix}
\]

numerics currently being run using latticeeasy...
AN ASIDE ON SCALES

MASSES

- fuzzy DM
- mass of fluctuation components
- limits from small scale crises - "catch 22 of WDM"
- latticeeasy first runs have shown bounds aren’t satisfied
- relaxing of couplings results in permissible abundance
STRINGY EMBEDDINGS

MONODROMIES || ALIGNMENT || ELSE?

Silverstein, Westphal; McAllister, Silverstein, Westphal 2008

Kaloper, Sorbo 2008

2πf

V(φ)

favoured UV completion?

Kim, Nilles, Peloso 2004
CONCLUSIONS AND OUTLOOK

- growth significant for detectable parameter ranges
- fluctuations radiate away abundance
- small scale crises can be eased, but not solved
- backreaction problem of monodromy unavoidable

- model ruled out from above *and* below? Watch this space!
CONCLUSIONS AND OUTLOOK

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THANKS FOR LISTENING AND HAPPY 4TH OF JULY!!