Compact T-branes

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Based on:
F.M., Savelli, Schwieger
1707.03797
Cooking a compactification

- Building a 4d string compactification can be seen as preparing a good meal: you need to add several ingredients that should not upset each other
  - Well understood 6d manifold $\rightarrow$ 4d EFT
  - Fluxes that fix the shape of the manifold
  - D-branes that add gauge interactions
- Depending on how elaborate you want your meal
  - SM sector + hidden sector
  - SUSY breaking
  - Inflationary sector

... you may need to add your ingredients in a more intricate way, growing the risk of affecting each other

Taken from Camara, Ibañez, Valenzuela '14
Key ingredient: BPS D-branes

- A **standard strategy** to build gauge sectors that respect each other is to consider superpositions of **mutually BPS gauge D-branes**.

- The better we know the **spectrum of BPS D-branes**, the richer the set of vacua than we can build from a certain compactification manifold.
Key ingredient: BPS D-branes

- A **standard strategy** to build gauge sectors that respect each other is to consider superpositions of **mutually BPS gauge D-branes**

- The better we know the **spectrum of BPS D-branes**, the richer the set of vacua than we can build from a certain compactification manifold

- Relatively benign class: **type IIB Calabi-Yau with O3/O7-planes**. Large volume $\rightarrow$ D3 & D7-branes

- D3-branes trivially BPS. **Kähler geometry** allows to build many examples of BPS D7-branes

- Similar statement for **F-theory compactifications**

*See e.g. talks by Taylor, Grimm, Cvetic…*
Type IIB/F-theory models

• In principle, to build a type IIB/F-theory model one may specify a set of four-cycles $S_a$ each of them wrapped by $N_a$ 7-branes, and a worldvolume flux $F$ on each of them

✦ A distant stack of 7-branes can give rise to a hidden SU(N) sector with a gaugino condensate

✦ Five 7-branes on the same four-cycle give rise to a SU(5) gauge group
  Typical choice for $S$: del Pezzo surface

✦ …
Type IIB/F-theory models

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- A distant stack of 7-branes can give rise to a hidden SU($N$) sector with a gaugino condensate.

- Five 7-branes on the same four-cycle give rise to a SU(5) gauge group.
  Typical choice for $S$: del Pezzo surface.

- ...

However, these are not all the most general BPS 7-branes. There are also T-branes.
Typically, one describes a 7-brane system with symmetry group $G$ in terms of two $\text{Adj}(G)$-valued two-forms on a four-cycle $S$. Considering both $F$ and $\Phi$ non-Abelian is the most general possibility.
What is a T-brane?

- Typically, one describes a 7-brane system with symmetry group $G$ in terms of two Adj$(G)$-valued two-forms on a four-cycle $S$.

T-branes are configurations such that $[\Phi, \Phi^\dagger] \neq 0$.

$F \rightarrow$ worldvolume flux

$\Phi \rightarrow$ displacement from $S$

Donagi, Katz, Sharpe'03
Hayashi et al. '09
Cecotti et al. '10
Donagi & Wijnholt'11

Recent formal developments:
Anderson, Heckman, Katz'13
Del Zotto et al. '14
Collinucci & Savelli'14
Collinucci et al. '16
Bena et al. '16
F.M & Schwieger'16
Mekareeya, Rudelius, Tomasiello'16
Anderson et al. '17
Bena, Blaback, Savelli'17
Collinucci, Giacomelli, Valandro'17

...
What is a T-brane?

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\[ \Phi, \Phi^\dagger \neq 0 \]

T-branes are configurations such that \([\Phi, \Phi^\dagger] \neq 0\).

\[ F \rightarrow \text{worldvolume flux} \]
\[ \Phi \rightarrow \text{displacement from S} \]

In F-theory GUTs, T-branes allow to obtain models with one family much heavier than the other two.

Chiou et al.'15
Font et al.'13
F.M., Regalado, Zoccarato'15
Carta, F.M., Zoccarato'15

Donagi, Katz, Sharpe'03
Hayashi et al.'09
Cecotti et al.'10
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In F-theory GUTs, T-branes allow to obtain models with one family much heavier than the other two.

In addition, proposal to use them for uplifting to de Sitter.

Cicoli, Quevedo, Valandro '15
T-branes and the Hitchin system

- The BPS equations for $F$ and $\Phi$ are generalisations of the Hitchin system

\[
\begin{align*}
F_{0,2}^0 &= 0 \\
\bar{\partial}_A \Phi &= 0
\end{align*}
\]

\[
\{ \text{F-term} \}
\]

\[
\omega \wedge F + \frac{1}{2} [\Phi, \bar{\Phi}] = 0 
\]

\[
\text{D-term}
\]
T-branes and the Hitchin system

- The BPS equations for $F$ and $\Phi$ are generalisations of the Hitchin system

\[
\begin{align*}
F^{0,2} &= 0 \\
\bar{\partial}_A \Phi &= 0
\end{align*} \right\} \text{F-term}
\]

\[
\omega \wedge F + \frac{1}{2} [\Phi, \bar{\Phi}] = 0 \quad \text{D-term}
\]

- For $\Phi$ proportional to the identity they reduce to

$\Phi$ holomorphic and $\ast F = - F$

easy to construct all possible examples given a Calabi-Yau $X_6$
T-branes and the Hitchin system

• The **BPS equations** for F and \( \Phi \) are generalisations of the **Hitchin system**

\[
\begin{align*}
F^{0,2} &= 0 \\
\bar{\partial}_A \Phi &= 0 \\
\omega \wedge F + \frac{1}{2} [\Phi, \bar{\Phi}] &= 0
\end{align*}
\]

**F-term**

**D-term**

• For general, **non-Abelian** \( \Phi \) there is a very **poor knowledge** of which BPS configurations does a compact manifold \( X_6 \) admit

• We will consider configurations over a **compact four-cycle** \( S \) such that \( \Phi \) is smooth, non-Abelian and without poles, and will put constraints on \( S \).

\[\downarrow\]

**Compact T-branes**
Recap

- One of the most developed areas in string theory model building is related to type IIB/F-theory compactifications at large volume.

- In such models, chirality is achieved through the presence of 7-branes wrapped on four-cycles, and worldvolume fluxes on them.

- For a single 7-brane the equations of motion select a holomorphic four-cycle with an anti-self-dual flux on it. Thanks to Kähler geometry, it is easy to describe the set of such 7-branes in a complex manifold $X_6$. This is to large extent why this type of model-building is so developed.

- When we consider several 7-branes together, it appears the possibility of non-Abelian embeddings via the matrix $\Phi$, also known as T-branes.

- There is so far no picture of the spectrum of T-branes in $X_6$, in particular because no global solutions have been constructed. The goal is to classify the spectrum of compact T-branes in a Kähler manifold $X_6$. 
Nilpotent T-branes

The simplest T-brane features $G=\text{SU}(2)$, $F$ Abelian and $\Phi^2=0$

\[
\begin{align*}
\mathbf{F} &= \begin{pmatrix} F & 0 \\ 0 & -F \end{pmatrix} \\
F &= F^h - i \partial \bar{\partial} g \\
\left[ \frac{F}{2\pi} \right] &= c_1(\mathcal{L}) \in H^{1,1}(S) \\
\Phi &= \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix} \\
[m] &\in H^{2,0}(S, \mathcal{L}^2)
\end{align*}
\]
T-branes and the Hitchin system

• The BPS equations for $F$ and $\Phi$ are generalisations of the Hitchin system

\[ \begin{align*}
F^{0,2} &= 0 \\
\bar{\partial}_A \Phi &= 0
\end{align*} \]

\[ \omega \wedge F + \frac{1}{2} [\Phi, \Phi] = 0 \]

F-term

D-term
Nilpotent T-branes

The simplest T-brane features $G=SU(2)$, $F$ Abelian and $\Phi^2=0$

$$F = \begin{pmatrix} F & 0 \\ 0 & -F \end{pmatrix}$$

$$F = F^h - i\partial\bar{\partial}g$$

$$\left[ \frac{F}{2\pi} \right] = c_1(\mathcal{L}) \in H^{1,1}(S)$$

$$\Phi = \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix}$$

$$[m] \in H^{2,0}(S,\mathcal{L}^2)$$

$$\omega \wedge F + \frac{1}{2} [\Phi, \bar{\Phi}] = 0$$

$$\Delta g = \varphi + c$$

$$\varphi = *[\Phi, \bar{\Phi}]\sigma_3$$

$$c = \frac{2}{\text{vol}(S)} \int_S F \wedge \omega$$
Nilpotent T-branes

We need to solve:

\[ \Delta g = \varphi + c \]

function \quad \text{function } \varphi > 0 \quad \text{constant} \]

on a compact four-cycle \( S \)

Necessary condition:

\[ \int_S \varphi + c = 0 \Rightarrow c < 0 \quad \text{(4d D-term)} \]

- Given this sign one can always find a solution by rescaling \( \Phi \) by a constant

- Finding an actual solution can be rather involved, because in general \( \varphi \) depends on the function \( g \), and so the above equation is non-linear
Nilpotent T-branes

Indeed recall that:

\[ \Phi = \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix} \quad [m] \in H^{2,0}(S, \mathcal{L}^2) \]

\[ \bar{\partial}_A \Phi = 0 \quad F = F^h - i\bar{\partial}\bar{\partial}g \]

This implies that:

\[ m = (\det h_0)^{1/2} e^g m^{\text{hol}} \, dx \wedge dy \]

h_0 is a metric whose two-form curvature is \( F^h \)

\[ \bar{\partial} \left( m^{\text{hol}} \, dx \wedge dy \right) = 0 \]

\[ m^{\text{hol}} \in H^0(S, \mathcal{L}^2 \otimes K_S) \]

\[ \varphi = \frac{\det h_0}{\sqrt{\det g_S}} |m^{\text{hol}}|^2 e^{2g} \]
Nilpotent T-branes

We need to solve:

\[ \Delta g = \varphi + c \]

function \[\text{function } > 0\]

constant

\[ c = \frac{2}{\text{vol}(S)} \int_S F \wedge \omega \]

\[ \varphi = *[\Phi, \Phi] \sigma_3 \]

on a **compact four-cycle** \( S \)

**Necessary condition:**

\[ \int_S \varphi + c = 0 \Rightarrow c < 0 \] (4d D-term)

- Given this sign one can always find a solution by rescaling \( \Phi \) by a constant.
- Finding an actual solution can be rather **involved**, because in general \( \varphi \) depends on the function \( g \), and so the above equation is **non-linear**.

\[ \Delta g = \left[ \frac{\det h_0}{\sqrt{\det g_S}} |m^{\text{hol}}|^2 \right] e^{2g} + c \]

**g-independent function**
The Hitchin Ansatz

Hitchin considered a similar problem for the Riemann surfaces. To find a solution he took the following Ansatz:

\[ \mathcal{L} \sim K_S^{-1/2} \quad \Rightarrow \quad \mathcal{M} = \mathcal{L}^2 \otimes K_S \quad \text{is trivial} \]

This implies that:

\[ m^{\text{hol}} \in H^0(S, \mathcal{M}) \quad \text{is constant} \]

\[ \det h_0 = \sqrt{\det g_S} \quad \text{(up to a conformal factor; equal for Kähler-Einstein)} \]

\[ \Delta g = |m^{\text{hol}}|^2 e^{2g} + c \quad \Rightarrow \quad g = 0 \]

(case when $S$ is Kähler-Einstein)
The Hitchin Ansatz

Hitchin considered a similar problem for the Riemann surfaces. To find a solution he took the following Ansatz:

\[ \mathcal{L} \simeq K_S^{-1/2} \implies \mathcal{M} = \mathcal{L}^2 \otimes K_S \text{ is trivial} \]

This implies that:

- \( m^{\text{hol}} \in H^0(S, \mathcal{M}) \) is constant
- \( \det h_0 = \sqrt{\det g_S} \) (up to a conformal factor; equal for Kähler-Einstein)

Finally:

\[ 0 > c = \frac{2}{\text{vol}(S)} \int_S F \wedge \omega = \frac{4\pi}{\text{vol}(S)} \int_S c_1(\mathcal{L}) \wedge \omega = -\frac{2\pi}{\text{vol}(S)} \int_S c_1(K_S) \wedge \omega = \frac{1}{\text{vol}(S)} \int_S \rho \wedge \omega \]

\[ \implies S \text{ is of negative curvature: } \rho = -\frac{|m^{\text{hol}}|^2}{2} \omega \]
Beyond Hitchin

This result could in principle be related to the Ansatz of Hitchin, who also only found solutions for Riemann surfaces of negative curvature.

One can generalise this Ansatz in a number of ways:

- **Non-Kähler-Einstein metric**
  \[ \Delta g = |m^\text{hol}|^2 e^{2(g-s)} + c \]

- **\( \mathcal{L} \not\propto K_S^{-1/2} \)**
  \[ \Delta g = \|m^\text{hol}\|^2_{\mathcal{M}} e^{2(g-s)} + c \]

- **Non-nilpotent** \( \Phi = \begin{pmatrix} 0 & m \\ p & 0 \end{pmatrix} \)
  \[ \Delta g = \left( \|m\|^2_{\mathcal{M}} e^{2g} - \|p\|^2_P e^{-2g} \right) e^{-2s} + c \]

Painlevé III-like equation
Beyond Hitchin

This result could in principle be related to the Ansatz of Hitchin, who also only found solutions for Riemann surfaces of negative curvature.

One can generalise this Ansatz in a number of ways:

- **Non-Kähler-Einstein metric**
  \[ \Delta g = |m^{\text{hol}}|^2 e^{2(g-s)} + c \]

- **\( L \not\cong K_S^{-1/2} \)**
  \[ \Delta g = \|m^{\text{hol}}\|^2_M e^{2(g-s)} + c \]

- **Non-nilpotent** \( \Phi = \begin{pmatrix} 0 & m \\ p & 0 \end{pmatrix} \)
  \[ \Delta g = (\|m\|^2_M e^{2g} - \|p\|^2_P e^{-2g}) e^{-2s} + c \]

**Question:** Do these solutions also need manifolds of negative curvature?
A no-go theorem

For a nilpotent T-brane: \( \Phi = \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix} \) \[ [m] \in H^{2,0}(S, \mathcal{L}^2) = H^0(S, \mathcal{M}) \]

By assumption, \( m \) exists and has no poles. This means that \( \mathcal{M} = \mathcal{L}^2 \otimes K_S \) corresponds to an effective divisor, of positive area.

\[
\int_S \omega \wedge c_1(\mathcal{M}) = \int_S \omega \wedge (2c_1(\mathcal{L}) + c_1(K_S)) \geq 0
\]

In addition, solving the 4d D-term equation implies that

\[
\int_S \omega \wedge c_1(\mathcal{L}) < 0
\]

Taking both into account we obtain

\[
\int_S \omega \wedge c_1(K_S) > 0 \quad \Rightarrow \quad \text{Rules out manifolds with vanishing or positive curvature}
\]
A no-go theorem (II)

The same holds for the non-nilpotent case:

\[
\Phi = \begin{pmatrix} 0 & m \\ p & 0 \end{pmatrix}
\]

\([m] \in H^{2,0}(S, L^2) = H^0(S, \mathcal{M})

\([p] \in H^{2,0}(S, L^{-2}) = H^0(S, \mathcal{P})

\]

\[c < 0 \implies 0 \leq \int_S \omega \wedge c_1(\mathcal{M}) < \int_S \omega \wedge c_1(K_S) < \int_S \omega \wedge c_1(\mathcal{P})\]

if \(c > 0\) the role of \(\mathcal{M}\) and \(\mathcal{P}\) is interchanged

Finally, one can show that the same is true for Lie groups \(G\) of higher rank. As long as the gauge connection \(A\) lies in the Cartan subalgebra.

\[\int_S \omega \wedge c_1(K_S) > 0 \implies \text{Rules out manifolds with vanishing or positive curvature}\]
Recap

- Solving the T-brane equations in a compact four-cycle $S$ impose constraints on it. Demanding that
  - $\Phi$ has no poles
  - $F$ is abelian

implies that the Ricci curvature of $S$ has a negative sign when projected into the Kähler form.

- In particular, $S$ cannot be K3 or a positive curvature four-cycle, like del Pezzo surfaces. Interestingly, these surfaces were proposed to build F-theory GUT models. We may still construct such T-brane backgrounds on four-cycles of indefinite or negative curvature.

- In both cases phenomena of wall-crossing may appear when we move in Kähler moduli space. Decay into non-mutually BPS components easy to engineer for the Hitchin Ansatz.

See S. Schwiger's talk
Conclusions

- **Type IIB/F-theory model building** at large volume encodes its richness in the possibility of having 7-branes wrapping four-cycles, and its power in the capability of constructing examples for them.

- Solutions for the **T-brane BPS equations** have been found in local patches, but a full understanding of T-branes over a compact four-cycle $S$ is missing.

- Here we have considered T-brane configurations over $S$ such that $\Phi$ has no poles and $F$ is Abelian.

- We have found the equations **cannot be solved** globally for four-manifolds of vanishing or positive curvature, like K3 or del Pezzo surfaces. This result also applies to $\Phi$’s that only have poles outside the T-brane sector, so that the worldvolume flux has no poles either.

- This implies that the simplest scheme for **F-theory GUT model-building may have to be revisited**.