Heterotic Wave Function Normalisation from Localization

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Outline

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Introduction

Yukawa couplings in 4d supergravity:

\[ K = K_{mod} + G_{IJ}(T, \bar{T}, S, \bar{S}, Z, \bar{Z}) \, C^I \bar{C}^J \]

\[ W = \lambda_{IJK}(Z) \, C^I C^J C^K \]

Metric \( G_{IJ} \) needs to be diagonalized for physical Yukawa couplings

\[ \rightarrow \text{Both } \lambda_{IJK} \text{ and } G_{IJ} \text{ are required for phenomenology} \]
Yukawa couplings in heterotic CY models:

Model specified by a CY manifold $X$ and a bundle $V \to X$.

Matter fields: $C \longleftrightarrow \nu \in H^1(X, V)$, harmonic

Holomorphic Yukawa couplings given by:

$$\lambda(\nu_1, \nu_2, \nu_3) = \int_X \Omega \wedge \nu_1 \wedge \nu_2 \wedge \nu_3$$

invariant under $\nu_i \to \nu_i + \bar{\partial} \alpha_i$

Holomorphic Yukawa couplings are quasi-topological: Harmonic representatives and CY metric not required
Matter field Kahler metric given by:

\[
G(\nu, \rho) = \frac{1}{V} \int_X \bar{\nu} \wedge (\ast_V \rho)
\]

not invariant under \( \nu \rightarrow \nu + \bar{\partial} \alpha \), \( \rho \rightarrow +\bar{\partial} \beta \)

Matter field Kahler metric is not quasi-topological.
Its calculation requires the CY metric and the HYM connection.

So far, only known method to work out \( G \): numerical

Donaldson 05, Headrick, Wiseman 05,
Douglas, Karp, Lucik, Reinbacher 06
Braun, Brelidze, Douglas, Ovrut 07
Anderson, Braun, Karp, Ovrut 10

disadvantages: 
- technically quite involved
- provides \( G \) at one point in moduli space
Main purpose of this talk: report progress on (approximate) analytic calculation of $G$ as a function of the moduli

But before....
Results for holomorphic Yukawa couplings

CY manifold: $X = \{p_\alpha = 0\} \subset A = \mathbb{P}^{n_1} \times \mathbb{P}^{n_2} \times \ldots$

co-dimensions of $X \subset A : k$

line bundle models:  

\begin{align*}
\text{bundle:} & \quad \mathcal{V} = \bigoplus_a \mathcal{L}_a \to A \\
\text{matter fields:} & \quad \hat{\nu}_a \in \Omega^a (A, \mathcal{V}) \\
& \quad a = 1, \ldots, d \leq k + 1
\end{align*}

$V = \mathcal{V}|_X = \bigoplus_a L_a \to X$

$\nu \in H^1 (X, V)$

``Wave function'' $\nu$ is determined by ambient space forms $\hat{\nu}_1, \ldots, \hat{\nu}_d$

Then, $\nu$ is said to be of type $d \in \{1, \ldots, k + 1\}$

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Vanishing property:

\[ d_1 + d_2 + d_3 < \dim(A) \quad \implies \quad \lambda(\nu_1, \nu_2, \nu_3) = 0 \]
Wave function normalization from localisation

Focus on a single line bundle $\mathcal{L} \rightarrow A$, $L = \mathcal{L}|_X \rightarrow X$ and type 1

ambient space $A$ : restriction $\rightarrow$ Calabi-Yau $X$:

Kahler form: $\hat{J} = t^i \hat{J}_i$ $\rightarrow$ $J = t^i J_i$ Ricci-flat
$[J] = [\hat{J}|_X]$

connection: $\hat{F} = -\frac{i}{2\pi} \partial \bar{\partial} \hat{H}$ $\rightarrow$ $F = -\frac{i}{2\pi} \partial \bar{\partial} H = k^i J_i$
HYM : $J^2 \wedge F \neq 0$

wave fct: $\hat{\nu} \in H^1(A, \mathcal{L})$ $\rightarrow$ $\nu \in H^1(X, L)$
$[\nu] = [\hat{\nu}|_X]$

$\nu$ harmonic: $\bar{\partial} \nu = 0$ $J \wedge J \wedge \partial (H \nu) = 0$
inner product:

\[ \langle \nu, \rho \rangle := \int_X \bar{\nu} \wedge (H \star \rho) = -\frac{i}{2} \int_X J \wedge J \wedge \bar{\nu} \wedge (H \rho) \]

Suppose that \( H|\nu|^2 \) is localised on a patch \( U \subset X \)

On \( U \) we can use (approximately) flat \( J \) and \( H \).
Does the wave function localise on projective spaces?

Toy example $\mathbb{P}^1$ with affine coordinate $z$, $\mathcal{L} = \mathcal{O}_{\mathbb{P}^1}(k)$, $k \leq -2$

Kahler form: $\hat{J} = \frac{i}{2\pi \kappa^2} dz \wedge d\bar{z}$  
$\kappa = 1 + |z|^2$

bundle metric: $\hat{H} = \kappa^{-k}$  

wave fct: $\hat{\nu} = \kappa^k \ P(\bar{z}) \ d\bar{z}$

$H|\nu|^2 \sim |P|^2 \kappa^k \sim |P|^2 e^{k|z|^2}$ for large $|k|$ localised near $z \simeq 0$
An explicit example

ambient space: $\mathcal{A} = \mathbb{P}^1 \times \mathbb{P}^3$

CY manifold: $X \sim \begin{bmatrix} \mathbb{P}^1 & 2 \\ \mathbb{P}^3 & 4 \end{bmatrix} \begin{array}{c} \hat{z}_1 \\ \hat{z}_2, \hat{z}_3, \hat{z}_4 \end{array}$

line bundle: $\mathcal{L} = \mathcal{O}_X(k_1, k_2)$, $k_1 \leq -2$, $k_2 > 0$

\[
\begin{align*}
\hat{J}_1 &= \frac{i}{2\pi} dz_1 \wedge d\bar{z}_1 \\
\hat{H} &= e^{-k_1 |z_1|^2 - k_2 \sum_{\alpha=2}^4 |z_\alpha|^2} \\
\hat{\nu} &= e^{k_1 |z_1|^2} \hat{P}(\bar{z}_1, z_\alpha) d\bar{z}_2.
\end{align*}
\]

$\hat{J} = t_1 \hat{J}_1 + t_2 \hat{J}_2$

2 Kahler moduli
defining eqn. for CY:

\[ p = p_0 + \sum_{a=1}^{4} p_a z_a + \mathcal{O}(\varepsilon^2) = 0 \quad \text{near} \quad U = \{ z_a \simeq 0 \} \]

How to choose \( J_i \)?

\[ J_i = \hat{J}_i \mid_U \quad \text{for} \quad U = \{ z_4 \simeq z_1 / \sqrt{6} \} \]

\[ \Rightarrow \quad J_i \wedge J_j \wedge J_k = -\frac{1}{16\pi^3} d_{ijk} \bigwedge_{a=1}^{3} dz_a \wedge d\bar{z}_a \]

\[ \Rightarrow \quad J^2 \wedge F \sim \mu(L) = 0 \quad \text{(HYM eqs locally satisfied)} \]

Families:

\[ \hat{\nu}_1 = e^{k_1 |z_1|^2} \bar{z}_1 I_1 \bar{z}_2 I_2 \bar{z}_3 I_3 \bar{z}_4 I_4 \ d\bar{z}_1 \]

\[ I_1 \in \{ 0, \ldots, -k_1 - 2 \} \quad I_2 + I_2 + I_3 \in \{ 0, \ldots, k_2 \} \]
Find harmonic wave fcts. $\nu_I$ with $[\nu_I] = [\hat{\nu}_I|_U]$ and work out

$$G_{I,J} = \frac{1}{V} \langle \nu_I, \nu_J \rangle$$

This leads to:

$$G_{I,J} = \frac{\mathcal{N}_{I,J}}{6t_1 + t_2}$$

with numbers

$$\mathcal{N}_{I,J} = \pi \frac{J_1! I_1! I_2! I_3! |k_1 + k_2/6|^{I_1-I_4+1} 6^{I_4/2+J_4/2+1}}{(I_1 - I_4)! |k_1|^{J_1+1}k_2^{I_2+I_3+2}} \theta(I_1 - I_4) \delta_{I_1-I_4,J_1-J_4} \delta_{I_2,J_2} \delta_{I_3,J_3}$$
Conclusion

• For heterotic line bundle models, we have a fairly good understanding of how to calculate holomorphic Yukawa couplings.

• For large flux, the matter field Kahler metric can be approximately calculated due to localisation.

• The local calculation can be linked to the global properties, so the result is obtained as a function of global moduli!

• Using localisation, we have calculated the matter field Kahler metric explicitly for simple examples.

• Analogous methods may well apply to F-theory in suitable global models.

• However, calculation becomes very involved for more complicated CY manifolds.
The global-local link allows us to better understand the limitations of this method. Some tensions emerge:

- The large flux required for localisation typically implies a large family number. Can the method be applied to realistic models?

- Large flux quickly runs up against anomaly cancellation.

Thanks