Quantifying the Scarcity of Weak Coupling Limits in F-theory Compactifications

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F-Theory

- F-theory is a strong coupling generalization of IIB string theory.
- Elliptically fibered CY fourfold X_4 over base $B_3 \leftrightarrow$ 4d effective theory.

$$y^2 = x^3 + fx + g\,.$$

- 7-branes on $\Delta = (4f^3 + 27g^2) = 0$.
- Sometimes can take $g_s \to$ small (WC) limit, to recover IIB on $B_3 = X_3/\sigma$.

Weak Coupling

- Weakly coupled type IIB: computability and control. Can be very useful to make this connection.
- F-theory: more general physics: gauge groups and matter, dark matter candidates, cosmology, etc. (Garcia-Etxebarria, Halverson, Nelson, Taylor, Grimm, Cvetic, Oehlmann, Raghuram, Mayorga, Otsuka, Choi, Lin, Weigand, Bies, Wang, Lawrie, Esole, Valandro,)
- Landscape questions: how often can one expect a connection with weakly coupled IIB? What percentage of bases give intrinsically strongly coupled physics? What is the obstruction to weak coupling?

Weak Coupling

Question

Which B_3 allow for a WC limit globally (no $O(1)$ g_s regions anywhere on B_3)? What physics (or geometry) obstructs weak coupling?

Computing the coupling

- Can compute g_s from inverting

$$J(q) = \frac{4f^3}{(4f^3 + 27g^2)} = \frac{1}{q} + 744 + 196884q + \dots .$$

$$q = e^{2\pi i \tau},\ \tau = c_0 + \frac{i}{g_s}$$

- $g_s \ll 1$ implies $\tau \to i\infty$, $J \to \infty$.

Sen's limit

- Sen's limit (See talks by Iñaki and Damian):

$$f = -3h^2 + \epsilon\eta, \quad g = -2h^2 + \epsilon h\eta - \frac{\epsilon^2\chi}{12}.$$

$$J = \frac{h^4}{\epsilon^2(\eta^2 - h\chi)}.$$

(Sen, Aluffi, Clingher, Donagi, Esole, Fullwood, Savelli, Wijnholt, Yau, . . .)

- $\epsilon \to$ small gives WC limit everywhere EXCEPT O7-plane locus $h = 0$, which gives a region of $\mathcal{O}(1)\, g_s$.

Global Weak Coupling

Need to cancel 7-brane charge locally to avoid any $\mathcal{O}(1)\, g_s$ regions!

Possible Fibers

Systematic: examine all possible fiber types (e.g. Halverson). Other than smooth or I_0^*, one finds strong coupling on given 7-brane stack, or on stack required for 7-brane charge cancellation.

F_i	l_i	m_i	n_i	Sing.	G_i	τ	g_s
I_0	≥ 0	≥ 0	0	none	none	$\mathbb{H}$	≥ 0
I_n	0	0	$n \geq 2$	A_{n-1}	$SU(n)$ or $Sp(\lfloor n/2 \rfloor)$	$i\infty$	0
II	≥ 1	1	2	none	none	$e^{2\pi i/3}$	$2/\sqrt{3}$
III	1	≥ 2	3	A_1	$SU(2)$	i	1
IV	≥ 2	2	4	A_2	$SU(3)$ or $SU(2)$	$e^{2\pi i/3}$	$2/\sqrt{3}$
I_0^*	≥ 2	≥ 3	6	D_4	$SO(8)$ or $SO(7)$ or G_2	$\mathbb{H}$	≥ 0
I_n^*	2	3	$n \geq 7$	D_{n-2}	$SO(2n-4)$ or $SO(2n-5)$	$i\infty$	0
IV^*	≥ 3	4	8	E_6	E_6 or F_4	$e^{2\pi i/3}$	$2/\sqrt{3}$
III^*	3	≥ 5	9	E_7	E_7	i	1
II^*	≥ 4	5	10	E_8	E_8	$e^{2\pi i/3}$	$2/\sqrt{3}$

Possible Fibers

Allowed Singularities for Global Weak Coupling

Only smooth and I_0^* in non-trivial weak coupling limit. Others have 7-brane backreaction that gives a region of $\mathcal{O}(1)\, g_s$, even when realized in orientifold models!

-Seiberg and Witten '95, Roan '96.

- This actually implies no monodromy $\rightarrow$ (geometric) gauge group is SO(8).
- We will say geometries with only smooth and I_0^* fibers at codimension-1 admit a non-trivial weak coupling limt (NTWCL).

Strategy

Which bases allow for a global weak coupling limit? $\rightarrow$ Which bases allow for only I_0^* and smooth fibers on all codimension-1 loci?

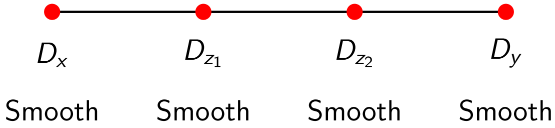
1. Start with a "minimal" smooth geometry that admit smooth elliptic fibration.
2. Do blowups to generate ensemble (crepant base changes).
3. Eventually blowups force $> I_0^*$ vanishing, no global weak coupling limit in these geometries.

Toric Bases

- In 1706.02299 $\sim 10^{755}$ toric bases for F-theory identified as blow-ups of smooth weak Fano TVs (Halverson, CL, Sung).
- Minimal geometries: smooth weak Fano TV $\leftrightarrow$ triangulated reflexive polytope Δ°.
- Before triangulation, singular toric variety with coordinate ring generated by the corners. Smoothing introduces exceptional divisors, with certain structure: modifies sections of $\mathcal{O}(-nK_B)$ mildly! Gives structure that we will use.
- Combinatorics: points $\leftrightarrow$ divisors, one-simplices $\leftrightarrow$ curves, two-simplices $\leftrightarrow$ points.

Blowups, NHC, and Weak Coupling

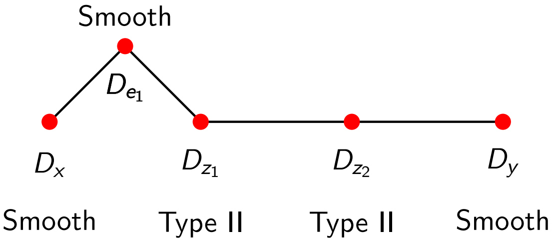
- Consider an edge of the 3d polytope. Points interior are exceptional divisors from resolution of singularity along curve $x = y = 0$. Generic Weierstrass model is smooth.



- "Ground" divisors are all height 1.

Blowups, NHC, and Weak Coupling

- Blowup interior to the edge, with height 2. This forces at least type *II* (non-Higgsable) on all internal points.



- For NTWCL we need to tune to I_0^* on internal points. This forces I_0^* on all points on edge!

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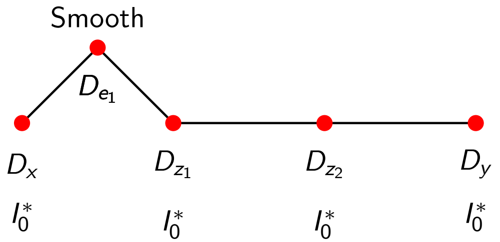
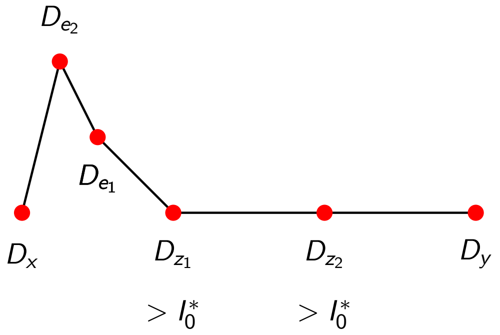


- For NTWCL we need to tune to I_0^* on internal points. This forces I_0^* on all points on edge!

BUs, NHC, and Weak Coupling

- Blowup again with height 3, forces $> I_0^*$ fiber. Further BUs will only force higher vanishing.



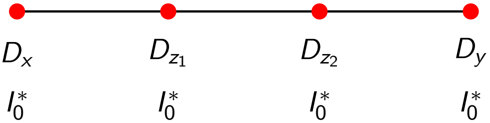
- Blowup with $h \geq 3 \rightarrow$ **no NTWCL**. Generalizes directly to blowups of points (trees over simplices).

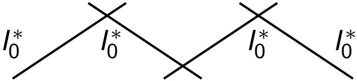
Toric Weak Coupling and Splitting I_0^*s

Toric NTWCL

Only weak Fano toric bases to admit NTWCL: FRST of reflexive polytopes, and height 2 blowups thereof. Probability $\sim 2 \times 10^{-379}$ in our ensemble!

- Simple physics: can tune I_0^*s on intersecting divisors, allowed base change simply separates intersecting I_0^*s.





Toric Weak Coupling and Splitting I_0^*s

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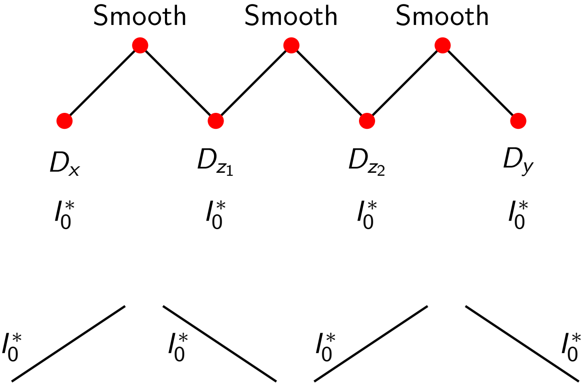

General Algebraic Bases

- Take inspiration from toric varieties: build up global geometry (Kähler threefold) from gluing local patches.
- Ensemble generated from crepant base changes, or blowups.
- Each patch: crepant resolution of orbifold $\mathbb{C}^3/G$, G finite subgroup of $SL(3, \mathbb{Z})$ (Reid).
- Possible options are:
  1. A-D-E singularities fibered over $\mathbb{C}$.
  2. Isolated singularities.
- A-type fibered over $\mathbb{C}$ and isolated singularities all admit crepant toric resolutions , so very easy to read off answer (Reid, Degeratu). However global geometry is not that of a toric variety!
- D and E type in progress.

General Algebraic Bases

- Consider local A_n singularity along $x = y = 0$.
- Local sections of $O(-4K_B)$ of the form $x^a y^b$.
- Resolve with blowups, introduce new exceptional coordinates z_i. Sections promoted to

$$x^a y^b z_1^{\alpha_1 a + (1-\alpha_1)b} \ldots z_1^{\alpha_n a + (1-\alpha_n)b} \, .$$

$$0 < \alpha_i < 1 \, .$$

- Implies same result as toric case.

Algebraic Base NTWCL

NTWCL limit only exists for height ≤ 2 BUs involving exceptional divisors. All other bases have $> I_0^*$ (non-Higgsable) vanishing along some divisor. Can only tune I_0^* on ground, and do BU to separate intersecting I_0^*s.

Conclusions

- NTWCL requires only I_0^* or smooth fibers, only gauge group $SO(8)$.
- Ensembles of CY fourfolds generated from blowups (crepant base changes) of some minimal geometries (toric and gluing of orbifold resolutions).
- NTWCL only when blowups involving exceptional divisors (interior to edges or facets) have $h \leq 2$.
- BUs correspond to separating intersecting I_0^* loci in base.
- Toric case: NTWCL occurs with probability $< 2 \times 10^{-379}$. General case expected to be just as small!
- Strong evidence that non-Higgsable 7-branes are totally generic in F-theory, and weak coupling limits are not.

Non-Trivial Weak Coupling Limit (NTWCL)

- Weak coupling limit is non-trivial if $g_s \ll 1$ everywhere on B_3, without $g_s = 0$.
- Need either I_0^* or smooth fibers on all codim-1 loci.

$$f = Fm^2, \quad g = Gm^3$$

$$m \in \Gamma(\mathcal{O}(-2K_B)), \quad F, G \in \Gamma(\mathcal{O}_B).$$

- This implies no monodromy $\rightarrow$ (geometric) gauge group is SO(8).
- Can generalize Sen's parametrization:

$$f = -3h^2 + \epsilon\eta$$

$$g = (-2 + \tilde{\epsilon})h^3 + \epsilon h\eta - \frac{\epsilon^2\chi}{12}$$

$$J = \frac{1}{\tilde{\epsilon}} .$$