The global gauge group structure of F-theory compactifications with U(1)s

based on arXiv:1706.08521 with Mirjam Cvetič

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Motivation

- Global gauge group structure of non-abelian symmetry given by Mordell–Weil torsion.
  [Apsinwall, Morrison '98], [Mayrhofer, Morrison, Till, Weigand '14]

- F-theory ‘Standard Models’ realize precisely physical spectrum under
  \( g_{SM} = su(3) \oplus su(2) \oplus u(1)_Y \) [LL, Weigand '14, '16], [Klevers et al '14], [Cvetič et al '15]
  \( \Rightarrow \) gauge group \([SU(3) \times SU(2) \times U(1)_Y]/\mathbb{Z}_6?\)

- What is the geometric origin?
Outline

1. Shioda map and the center of gauge groups
2. Example: F-theory ‘Standard Model’
3. Global group structure as charge constraints
4. Conclusions & outlook
Review: Shioda map and \( u(1) \)s in F-theory

- Sections \( \sigma_k \) of smooth elliptic Calabi–Yau \( \pi : Y \to B \) form Mordell–Weil (MW) group:
  \[
  \text{MW}(Y) = \mathbb{Z}^m \times \prod_t \mathbb{Z}_{k_t}. \quad [\sigma_k] = S_k, \quad k = 0, \ldots, m \text{ are independent integer divisors}.
  \]

- Codim. 1 singular fibers \( \Rightarrow E_i \) (Cartan divisors of non-ab. algebra \( \mathfrak{g} \)); Shioda–Tate–Wazir:
  \[
  H^{1,1}(Y, \mathbb{Q}) = \langle S_1, \ldots, S_m \rangle \oplus \langle S_0, E_i \rangle \oplus \pi^* H^{1,1}(B, \mathbb{Q}).
  \]

- \( \exists \) injective homomorphism (Shioda map) \( \varphi : \text{MW}(Y) \to H^{1,1}(Y, \mathbb{Q}) \), s.t. image is ‘orthogonal’ to \( \langle S_0, E_i \rangle \oplus H^{1,1}(B, \mathbb{Q}) \).

- General form: \( \varphi(\sigma) = \lambda (S - S_0 + \sum_i l_i E_i (+D_B)) \), \( \lambda \) a priori not constrained; for now, fix \( \lambda = 1 \).

- \( u(1) \) gauge field \( A \) arise from KK-reduction \( C_3 = A \wedge \varphi(\sigma) + \ldots \)
Fractional $u(1)$ charges

- $u(1)$ charge of matter states from codim. 2 fibral curves $\Gamma$ given by $\Gamma \cdot \varphi(\sigma)$ with $\varphi(\sigma) = S - S_0 + \sum_i l_i E_i$.

- Coefficients $l_i$ are determined by ‘orthogonality’ of $\varphi(\sigma)$ with $E_i = (\mathbb{P}^1_i \to \{\theta\})$; explicitly, $l_i = \sum_j (C^{-1})_{ij} (S - S_0) \cdot \mathbb{P}^1_j$, with $C_{ij} = -E_i \cdot \mathbb{P}^1_j$. 

$\Gamma$-w, v fibral curves with weights $w, v$ in the same $g$-rep $\Rightarrow \Gamma_v = \Gamma_w + \beta_k P^1_k$ with $\beta_k \in \mathbb{Z} \Rightarrow l_{i v} = l_i (E_i \cdot \Gamma_v) = l_i (E_i \cdot \Gamma_w - \beta_k C_{ik}) = l_{i w} - \beta_k (S - S_0) \cdot P^1_k \in \mathbb{Z}$.
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- $l_i$ are in general fractional, depend on the fibre split type and gauge algebra $\mathfrak{g}$. However, there is always an integer $\kappa$ s.t. $\forall i : \kappa l_i \in \mathbb{Z}$.

$\implies \kappa \varphi(\sigma)$ has manifestly integer class.
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Non-trivial central element from Shioda map

(a) There is $\kappa \in \mathbb{N}$ s.t. $\forall i : \kappa l_i \in \mathbb{Z} \implies q_{u(1)} = \frac{n}{\kappa}, n \in \mathbb{Z}$ (pick smallest such $\kappa$).

(b) Two weights $w, v$ in the same $g$-rep $\mathcal{R}_g$: $l_i w_i = l_i v_i \mod \mathbb{Z} =: L(\mathcal{R}_g) \implies \kappa L(\mathcal{R}_g) \in \mathbb{Z}$.

Construct non-trivial central element of $U(1) \times G$:
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  $\implies q(w) - l_i (E_i \cdot \Gamma) = q(w) - l_i w_i =: \xi(w) = (S - S_0) \cdot \Gamma \in \mathbb{Z}$. 

C := \left[ e^{2\pi i q(w)} \otimes (e^{-2\pi i l_i w_i} \times 1) \right] w

= \left[ e^{2\pi i \xi(w)} \otimes (e^{-2\pi i L(\mathcal{R}_g)} \times 1) \right] w

defines element in centre of $U(1) \times G$; (a) $\implies C_\kappa = 1$.
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- But also: \( C(w) = \exp(2\pi i \xi(w))w = w \).
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$\implies G_{\text{global}} = \frac{U(1) \times G}{\langle C \rangle} \cong \frac{U(1) \times G}{\mathbb{Z}_{\kappa}}$
Toric construction with gauge algebra $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$. [Klevers et al '14], [Cvetič et al '15]

- $\varphi(\sigma) = S - S_0 + \frac{1}{2} E_{1}^{\mathfrak{su}(2)} + \frac{1}{3} (2 E_{1}^{\mathfrak{su}(3)} + E_{2}^{\mathfrak{su}(3)}) \Rightarrow C^6 = 1$,
- so $G_{\text{global}} = [SU(3) \times SU(2) \times U(1)]/<C> \cong [SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$.

<table>
<thead>
<tr>
<th>$\mathcal{R}_{\mathfrak{su}(3) \oplus \mathfrak{su}(2)}$</th>
<th>$(3, 2)$</th>
<th>$(3, 1)$</th>
<th>$(1, 2)$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$L(\mathcal{R})$</td>
<td>$1/6$</td>
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geometrically realized matter: $(3, 2)_{1/6}$, $(1, 2)_{-1/2}$, $(3, 1)_{2/3}$, $(3, 1)_{-1/3}$, $(1, 1)_{1} = (\text{physical})$ Standard Model representations.
Example: F-theory ‘Standard Model’

Toric construction with gauge algebra $su(3) \oplus su(2) \oplus u(1)$. [Klevers et al '14], [Cvetič et al '15]

- $\varphi(\sigma) = S - S_0 + \frac{1}{2} E_{su(2)}^{\text{su}(2)} + \frac{1}{3} (2 E_{su(3)}^{\text{su}(3)} + E_{su(3)}^{\text{su}(3)}) \Rightarrow C^6 = 1,$

so $G_{\text{global}} = [SU(3) \times SU(2) \times U(1)]/\langle C \rangle \cong [SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$.

$$
\begin{array}{c|c|c|c|c}
\mathcal{R}_{su(3) \oplus su(2)} & (3, 2) & (3, 1) & (1, 2) & (1, 1) \\
L(\mathcal{R}) & 1/6 & 2/3 & 1/2 & 0 \\
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Similar situation in models with $su(3) \oplus su(2) \oplus u(1)_1 \oplus u(1)_2$ [LL, Weigand '14, '16].

After identifying hypercharge: $G_{\text{global}} = \frac{SU(3) \times SU(2) \times U(1)_Y \times U(1)_\perp}{\mathbb{Z}_6 \times \mathbb{Z}_\kappa}$.
Global group structure as charge constraints

- $\sigma$ torsional $\Rightarrow \varphi(\sigma) = 0$ (no $u(1)$), global group structure $G/\mathbb{Z}_\kappa$.
  $\implies$ not all $g$-reps allowed. [Mayrhofer, Morrison, Till, Weigand '14]

- $\sigma$ free, then global group structure $[U(1) \times G]/\mathbb{Z}_\kappa \Rightarrow u(1)$ charges of $g$-reps constrained:
  For $\mathcal{R}^{(i)} = (q^{(i)}, \mathcal{R}^{(i)}_g)$ we have $q^{(i)} = L(\mathcal{R}^{(i)}_g) \mod \mathbb{Z}$.
  For $g = su(5)$: [Braun, Grimm, Keitel '13], [Lawrie, Schäfer-Nameki, Wong, '15]

Argument derived with normalization $\lambda = 1$ for Shioda map $\Rightarrow$ ‘preferred’ charge normalization in F-theory: can read off global gauge group from fractional $u(1)$ charges.

Equivalently:
MW-group finitely generated $\longrightarrow$ global gauge group structure, refined charge quantization.
An F-theory ‘swampland’ criterion

- In preferred normalization, singlets \((E_i \cdot \Gamma = 0)\) have integral \(u(1)\) charges. Observation: in all \(u(1)\)-models with matter, smallest singlet charge is 1. \(\implies\) use singlets as ‘measuring stick’ for charges.

- Necessary condition for field theory to be F-theory compactification: normalize \(u(1)\) such that all singlets charges are as above

\[ \Rightarrow \text{for all matter } R^{(i)} = (q^{(i)}, R^{(i)}_g) \text{ we have } q^{(i)} - L(R^{(i)}_g) \in \mathbb{Z}. \]  
This implies:

1. If \(R^{(1)} = (q^{(1)}, R^{(1)}_g)\) and \(R^{(2)} = (q^{(2)}, R^{(2)}_g)\), then \(q^{(1)} - q^{(2)} \in \mathbb{Z}\).
2. If \(\bigotimes_{i=1}^n R^{(i)}_g = 1_g \oplus \ldots\), then \(\sum_{i=1}^n q^{(i)} \in \mathbb{Z}\).
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- Necessary condition for field theory to be F-theory compactification: normalize \( u(1) \) such that all singlets charges are as above
  \( \Rightarrow \) for all matter \( R^{(i)} = (q^{(i)}, R_g^{(i)}) \) we have \( q^{(i)} - L(R_g^{(i)}) \in \mathbb{Z} \). This implies:
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  2. If \( \otimes_{i=1}^{n} R_g^{(i)} = 1_g \oplus ... \), then \( \sum_{i=1}^{n} q^{(i)} \in \mathbb{Z} \).

- Problem: What if no singlets? (Non-higgsable \( u(1) \) in 6D without matter [(Martini), Morrison, (Park), Taylor ’14, ’16, [Wang ’17]) Situation in 4D?

- Other manifestations of the preferred normalization in field theory?
Conclusions & outlook

- Shioda map of sections ⇒ non-trivial global gauge group structure of $u(1) \oplus g$ theory. Example: F-theory explicitly realizes Standard Model $[SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$.

- Equivalently: refined charge quantization condition. In ‘preferred’ charge normalization:
  1. Charge lattice of $R_g$-matter integer spaced despite fractional charges.
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- Possible F-theory swampland criterion?

- What about charges of massive states? Spectrum of non-local operators?

- Global gauge group structure from higgsing (see paper) ⇒ some unhiggsed model with non-minimal codim. 2 loci. Physical explanation?
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Thank you!