F-theory and AdS_3/CFT_2

Craig Lawrie

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1612.06393 with S. Schäfer-Nameki, T. Weigand
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AdS_3/CFT_2 and F-theory

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F-theory is physics in terms of geometry

$$\begin{array}{ccccc} \mathbb{E}_\tau & \hookrightarrow & Y & & \\ & & \downarrow & & \\ & & B & \supset & C \end{array}$$

F-theory is physics in terms of geometry

Talks at this conference:

SO(10) Fibers [Oehlmann]
E6 Fibers [Garcia-Etxebarria]
SO(8) Fibers [Long]
MSSM[Mayorga]
(4,6) Curves [Sung]
→ $\mathbb{E}_\tau$

$$\mathbb{E}_\tau \hookrightarrow Y$$
$$\downarrow$$
$$B \supset C$$

Global Anomalies [Corvilain, Greiner, Grimm]
Sheaf Cohomology [Bies, Weigand]
Rational Sections [Cvetic, Lin]
T-branes [Marchesano, Schweiger]
Periods [Otsuka]
Euler Characteristics [Esole]
Matrix Factorisations [Valandro]
→ Y

Singular Divisors [Raghuram]
Possible Bases [Halverson, Nelson, Taylor, Wang]
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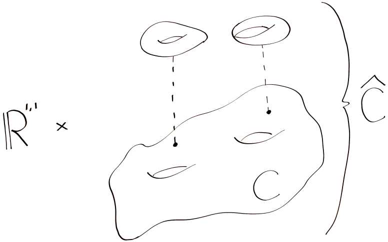
C ← Curves [This Talk]

Strings from D3-branes on Curves

D3-branes can wrap curves in base of F-theory

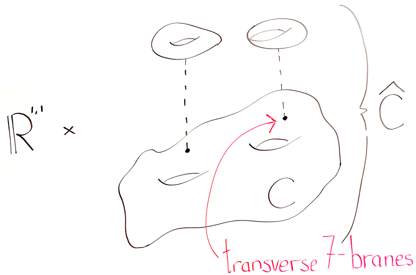


D3-branes on $\mathbb{R}^{1,1} \times C$

These are strings in D dimensions

D	
6	self-dual strings
4	dual to (half-SUSY) instantons; cosmic strings
2	spacetime filling, necessary for tadpole cancellation

The Principle Feature of F-theory



degenerations of fiber

$\Rightarrow$ transverse 7-branes

$\Rightarrow$ axiodilaton monodromy on C

$\to \mathcal{N} = 4$ SYM with varying coupling, τ, on $C \subset$ D3-brane

For $d\tau = 0$ worldvolume theory on string is σ-model into Hitchin moduli space [Bershadsky, Johanson, Sadov, Vafa]

For $d\tau \neq 0$, what is SCFT on string?

Roadmap

1. Single D3-brane on C with varying τ [CL, Schäfer-Nameki, Weigand]
   → study explicitly via topological duality twist
2. Multiple D3-branes on C with varying τ
   [Couzens, CL, Martelli, Schäfer-Nameki, Wong]
   → no explicit construction
   → construct AdS_3 supergravity duals
   → determine central charges from holography
3. Crosscheck [Couzens, CL, Martelli, Schäfer-Nameki, Wong]
   → central charges from M-theory via M/F-duality
   → central charges from microscopic constructions
      → self-dual strings in 6d and M5-brane anomaly inflow

Topological Duality Twist

Abelian $\mathcal{N} = 4$ SYM

$\Rightarrow$ "bonus" $U(1)_D$ symmetry [Intriligator], [Kapustin, Witten]

$$\gamma : \tau \to \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}) \quad \to \quad e^{i\alpha(\gamma)} \equiv \frac{c\tau + d}{|c\tau + d|} \in U(1)_D$$

Objects have charge q_D if transforms by $e^{iq_D\alpha(\gamma)}$ under γ

Object	F^+	F^-	Ψ, Q	$\widetilde{\Psi}, \widetilde{Q}$	ϕ_i
q_D	1	-1	$1/2$	$-1/2$	0

We have a $U(1)_D$ connection

$$\mathcal{A}_D = \frac{\mathrm{d}\tau_1}{2\tau_2}$$

Topological duality twist: To preserve SUSY compensate non-trivial transformation of supercharges under holonomy of C and $U(1)_D$ by R-symmetry transformation. [Martucci]

Central Charges

Construct topological duality twisted dimensional reduction to 2d

[CL, Schäfer-Nameki, Weigand]

SUSY on worldvolume of string depends on dimension:

D	8	6	4	2
SUSY	$(0, 8)$	$(0, 4)$	$(0, 2)$	$(0, 2)$

Can compute the central charges in each case; for F-theory to 6d:

$$c_R = 3C \cdot C + 3c_1(B) \cdot C$$

$$c_L = 3C \cdot C + 9c_1(B) \cdot C$$

How to generalize to multiple D3-branes on C?

Topological duality twist does **not** (obviously) generalize

→ instead can consider M5-branes [Assel, Schäfer-Nameki]

Can consider AdS/CFT ⇒ large N ⇒ large numbers of D3-branes

Enter AdS

Explore $\texttt{CFT}_d$ via AdS_{d+1} solutions of gravity [Maldacena]

Top-down approach: construct general supersymmetric solutions of Type II/11d SUGRA with AdS_{d+1} factor [Martelli, Sparks]

For F-theory: Type IIB solutions with AdS_{d+1} and non-trivial τ
- $\rightarrow$ τ variation comes from 7-branes; log-singularities and monodromy
  - $\rightarrow$ no such solutions known with full $SL(2, \mathbb{Z})$ monodromy
    - $\rightarrow$ for poles in τ see [Couzens], [D'Hoker,Gutperle, Uhlemann]

AdS_3 and 5d Black Holes

AdS_3 arises generally as the near horizon limit of black holes in 5d

Mircostate counting of dual CFT_2
→ microscopic origin of black hole entropy
→ with enough SUSY can(?) compute exact degeneracies of states

5d BPS black holes arise from 6d BPS strings on S^1
→ microstate counting of strings in 6d → macroscopic entropy

In 5d supergravity entropy from string microstates done with

- $\mathcal{N} = 4$ or $\mathcal{N} = 2$ [Strominger, Vafa], [Breckenridge, Myers, Peet, Vafa]
- $\mathcal{N} = 1$ [Vafa], [Haghighat, Murthy, Vafa, Vandoren]

In [Haghighat, Murthy, Vafa, Vandoren] entropy determined for $N = 1$ via topological duality twist and effective 6d supergravity

General Solutions for IIB with AdS_3 Factor and $(0,2)$ SUSY

IIB content:

$$F_5 \longleftrightarrow \text{D3-branes}$$

$$G_3 \begin{cases} F_3 & \longleftrightarrow \text{D1/D5-branes} \\ H_3 & \longleftrightarrow \text{F-strings/NS5-branes} \end{cases}$$

$$\tau = C_0 + ie^{-\Phi} \longleftrightarrow \text{7-branes}$$

Set $G_3 = 0$

General starting point:

$$ds^2 = e^{2A} ds^2(\text{AdS}_3) + ds^2(M_7)$$

$$F_5 = (1 + *)\text{vol}(\text{AdS}_3) \wedge F^{(2)}$$

To preserve $(0,2)$ SUSY solve Killing spinor equation

$$\nabla_M \epsilon + \frac{i}{192} \Gamma^{P_1 P_2 P_3 P_4} F_{M P_1 P_2 P_3 P_4} \epsilon = 0$$

General Solutions for IIB with AdS_3 Factor and $(0,2)$ SUSY

General solution [Couzens, CL, Martelli, Schäfer-Nameki, Wong]

$$\begin{array}{ccc} S^1 & \hookrightarrow & M_7 \\ & & \downarrow \\ & & M_6 \end{array}$$

S^1 fibration provides $U(1)_r$ R-symmetry of $(0,2)$

τ varation combines into an auxilliary Kähler elliptic fibration M_8 over M_6 with non-trivial constraint

$$\Box_8 R_8 - \frac{1}{2} R_8^2 + R_{8ij} R_8^{ij} = 0$$

First consider more SUSY

→ $(0,4)$ SUSY ⇒ dual to strings in 6d

→ $(2,2)$ SUSY

Preserving $(0,4)$ SUSY

Requiring $(0,4)$ is highly constrained, $A = \text{const}$ and

$$\begin{array}{ccccccc} S^1 & \hookrightarrow & S^3 & & Y_3 & \hookleftarrow & \mathbb{E}_\tau \\ & & \downarrow & & \downarrow & & \\ M_6 = & & S^2 & \times & B_2 & & \end{array}$$

Killing spinors transform in $(\mathbf{2},\mathbf{1})$ of S^3 isometry

$$SO(4) = SU(2)_r \times SU(2)_L$$

$$SU(2)_r \to \text{superconformal R-symmetry}$$
$$SU(2)_L \to \text{additional flavour symmetry}$$

Preserving $(0,4)$ SUSY

Requiring $(0,4)$ is highly constrained $A = \text{const}$ and

$$\begin{array}{ccccccc} S^1 & \hookrightarrow & S^3/\Gamma & & Y_3 & \hookleftarrow & \mathbb{E}_\tau \\ & & \downarrow & & \downarrow & & \\ M_6 = & & S^2 & \times & B_2 & & \end{array}$$

Killing spinors transform in $(\mathbf{2},\mathbf{1})$ of S^3 isometry

$$SO(4) = SU(2)_r \times SU(2)_L$$

$$\begin{aligned} SU(2)_r &\to \text{superconformal R-symmetry} \\ SU(2)_L &\to \text{additional flavour symmetry when } \Gamma = 1 \end{aligned}$$

We preserve the same SUSY for $\Gamma \subset SU(2)_L$ finite subgroup.

$(0, 4)$ Solution

General F-theory solution of Type IIB SUGRA dual to 2d $(0, 4)$ is

$$\begin{array}{ccc} & \mathbb{E}_\tau \hookrightarrow & Y_3 \\ & & \downarrow \\ \mathrm{AdS}_3 \times S^3/\Gamma \times & & B_2 \end{array}$$

with F_5 flux

$$F_5 = (1 + *)\mathrm{vol}(\mathrm{AdS}_3) \wedge J_B$$

J_B is Kähler form on B Poincaré dual to a curve C

$\Rightarrow C$, wrapped by D3-brane, ample in B

Kaluza–Klein Monopoles

$$\text{Take } \Gamma = \mathbb{Z}_M$$

$S^3/\mathbb{Z}_M$ is near horizon of Taub-NUT metric; brane solution is

$$\mathbb{R}^{1,1} \times TN_M \times B_2$$

with N D3-branes on $\mathbb{R}^{1,1} \times C$
$\rightarrow$ near-horizon

$$\text{AdS}_3 \times S^3/\mathbb{Z}_M \times B_2$$

$$M \text{ Kaluza–Klein monopoles on } \mathbb{R}^{1,1} \times B_2$$

$M = 1$ is a special case: near-horizon geometry is the same for zero or one KK monopoles

Brown–Henneaux formula

$$c = \frac{3R_{\text{AdS}}}{2G_N^{(3)}}$$

$$c^{\text{IIB}}_{\text{SUGRA}} = N^2 \frac{3\text{vol}(S^3/\mathbb{Z}_M)\text{vol}(B)32\pi^2}{\text{vol}(S^3/\mathbb{Z}_M)} = 6N^2 M \text{vol}(B)$$

Further

$$\text{vol}(B) = \int_B J_B \wedge J_B = C \cdot C$$

So

$$c^{\text{IIB}}_{\text{SUGRA}} = 3N^2 M C \cdot C$$

is the leading order contribution to the (left and right) central charge.

Gravitational Chern–Simons couplings from 7-branes bulk

$$S_{CS}(\Gamma_{\mathrm{AdS}_3}) = \frac{c_L - c_R}{96\pi} \int_{\mathrm{AdS}_3} \omega_{CS}(\Gamma_{\mathrm{AdS}_3})$$

$\Rightarrow$

$$c_L - c_R = 6Nc_1(B) \cdot C$$

Gauging $SO(4)$ isometry of S^3

$\Rightarrow$

$$k_r^{(1)} = \frac{1}{2} N c_1(B) \cdot C$$

Central Charges from Type IIB SUGRA

Leading and subleading central charges

$$c_R^{\text{IIB}} = 3N^2 C \cdot C + 3N c_1(B) \cdot C$$
$$c_L^{\text{IIB}} = 3N^2 C \cdot C + 9N c_1(B) \cdot C$$

Matches with spectrum computation for $N = 1$:

$$c_R^{\text{spectrum}} = 3C \cdot C + 3c_1(B) \cdot C$$
$$c_L^{\text{spectrum}} = 3C \cdot C + 9c_1(B) \cdot C$$

Only for $M = 1$

$\Rightarrow$ subleading contributions for $M > 1$ tricky

$\Rightarrow$ look at T-duality to M-theory

Recap – The Story So Far

1. Constructed general solution of Type IIB supergravity with
   $\rightarrow$ $(0,4)$ SUSY in dual SCFT
   $\rightarrow$ $G_3 = 0$ and arbitrary τ
2. Geometry:
$$\mathrm{AdS}_3 \times S^3/\Gamma \times B_2$$
3. Flux through (ample) curve in $B_2 \Rightarrow N$ D3-branes on C
4. Dual SCFT
   $\rightarrow$ worldvolume theory of string in 6d F-theory compactification

T-duality to M-theory

F-theory on Y_3 T-dual to M-theory on Y_3

General solution:

$$\mathrm{AdS}_3 \times S^2 \times Y_3$$

with flux

$$G_4 = \mathrm{dvol}(S^2) \wedge J_{Y_3}$$

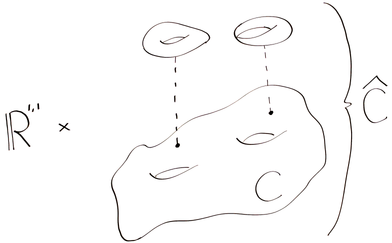
(See [Colgain, Wu, Yavartanoo])

J_{Y_3} is Kahler form on Y_3 Poincaré dual to divisor

$$MB + N\widehat{C}$$

T-duality to M-theory

M5-branes on $\mathbb{R}^{1,1} \times P$
$P \in |MB + N\widehat{C}|$



$$N \text{ D3-branes on } C \longleftrightarrow N \text{ M5-branes on } \widehat{C}$$
$$M \text{ KK monopoles } \longleftrightarrow M \text{ M5-branes on } B$$

See also [Bena, Diaconescu, Florea]

$\widehat{C} \cdot \widehat{C} \cdot \widehat{C} = 0$
$\Rightarrow$ divisor $\widehat{C}$ not ample, not Poincaré dual to Kähler form
$\Rightarrow$ no AdS dual to string from M5-branes wrapping $\widehat{C}$

Central Charges from M-theory

KK monopoles now M5-branes on B

→ Brown–Henneaux for holographic central charges for all M

$$c_R^{\text{M-th}} = 3N^2 M C \cdot C + 3N(2 - M^2) c_1(B) \cdot C$$
$$c_L^{\text{M-th}} = 3N^2 M C \cdot C + 3N(4 - M^2) c_1(B) \cdot C$$

Matches $c_{R,L}^{\text{IIB}}$ for $M = 1$

Includes both leading and subleading orders in N

→ also subsubleading → center of mass contributions

→ not discussed today (but agrees with microscopic constructions)

D3-branes on C

$\rightarrow$ self-dual strings (coupled to self-dual 2-form B) in 6d

$\rightarrow$ anomaly polynomial known [Berman, Harvey], [Shimizu, Tachikawa]

see also [CL, Schäfer-Nameki, Weigand]

$$I_4^{\text{SDS}} = -\frac{1}{24} p_1(T) \left[6 N c_1(B) \cdot C \right] + c_2(R) \left[\frac{1}{2} N^2 C \cdot C + \frac{1}{2} N c_1(B) \cdot C \right] + \cdots$$

Superconformal algebra relation

$$c_R = 6 k_R = 3 N^2 C \cdot C + 3 N c_1(B) \cdot C$$

Gravitational anomaly

$$c_L - c_R = 6 N c_1(B) \cdot C$$

Anomaly polynomial for N M5-branes [(Freed,) Harvey, Minasian, Moore]

$$I_8[N] = N I_8[1] + \frac{1}{24}(N^3 - N) p_2(\mathcal{N})$$
$$I_8[1] = \frac{1}{48}\left[p_2(\mathcal{N}) - p_2(TW) + \frac{1}{4}(p_1(TW) - p_1(\mathcal{N}))^2\right]$$

Integrate over complex surface P (say $MB + N\widehat{C}$)

$$I_4^{\mathrm{M5}}[P] = -\frac{1}{24} p_1(W_2)\left[\frac{1}{2} c_2(Y_3) \cdot P\right] + p_1(\mathcal{N}_3)\left[\frac{1}{6} P \cdot P \cdot P\right] + \cdots$$

Central charges ($P = MB + N\widehat{C}$ ample) [Maldacena, Strominger, Witten]

$$c_R = 6k_3 = 3N^2 MC \cdot C + (2 - M^2) N c_1(B) \cdot C$$
$$c_L - c_R = 2N(4 - M^2) c_1(B) \cdot C$$

Summary of $(0,4)$ Solution

Construct general AdS_3 solution of IIB SUGRA with dual $(0,4)$ SCFT

Computed holographic central charges $(M=1)$

$$c_R = 3N^2 C\cdot C + 3Nc_1(B)\cdot C$$
$$c_L = 3N^2 C\cdot C + 9Nc_1(B)\cdot C$$

Agrees with central charge computation from

1. 11d supergravity
2. Self-dual strings in 6d
3. M5-brane anomaly inflow
4. Spectrum (for $N=1$)

Conclusions and Future Directions

- Started systematically exploring holographic constructions in F-theory – varying axio-dilaton.
- Constructed AdS_3 solutions preserving $(0,2)$, $(0,4)$, and $(2,2)$ SUSY in dual CFT_2
  $\rightarrow$ what are the dual CFTs for $(0,2)$, $(2,2)$?
- For $(0,4)$ we obtained a microscopic understanding of the holographic constructions
  $\rightarrow$ what about $G_3 \neq 0 \rightarrow$ all AdS_3 solutions dual to $(0,4)$
- AdS duals to strings of minimial 6d SCFTs [del Zotto, Lockhart]
  $\rightarrow$ curve wrapped by D3-branes not ample