Fourier Mukai Transformation of Vector Bundles over Blown Up spaces

String Phenomenology
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• Motivations

- Fourier Mukai transform describe the moduli space of stable vector bundle as a fibration

$$\mathcal{Y} \to \mathcal{M}$$

$\mathcal{M}$: Moduli space of some n-sheeted cover of the base

$\mathcal{Y}$: Moduli space of a line bundle over that cover

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- It is possible compute the cover, but the line bundle over that is not well understood.

Bershadsky et. al, hep-th/9712023

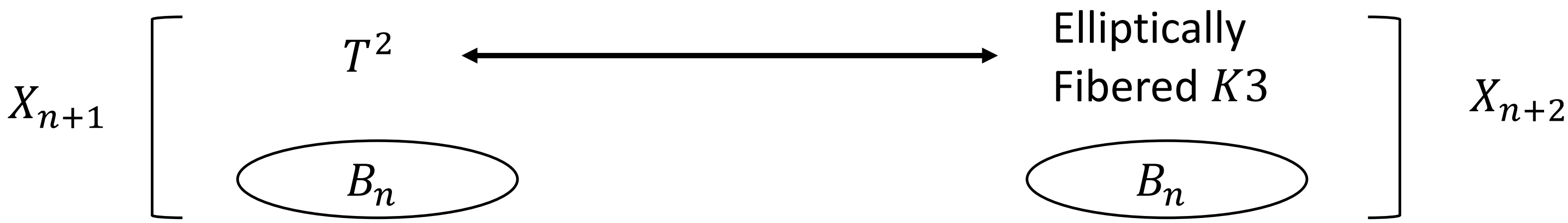
❖ The obstacle is, limited information about the divisors inside the n-sheeted cover.

- Previous works can be somewhat extended by studying the contribution of the induced divisors on the spectral cover from exceptional divisors in the heterotic CY.

• Het/F Duality Review

Vafa 96, Evidence for F-theory

Heterotic E8 × E8 F-Theory



- Volume of B_n shoude be large relative to the torus, (Adiabatic Approximation)

Vafa, Witten hep-th/9507050

- $N = 1$ SUSY in effective theory requires,
  - X_{n+1} and X_{n+2} are both CY
  - V is Stable
- $C_1(V) = 0$ for subgroups of E8

• Spectral Cover Construction

FMW hep-th/9701162
FMW alg-geom/9707004

Theorem (HN): V_{E_p} is semistable for a generic elliptic curve.

Theorem (Atiyah 57): a semistable vector bundle over a smooth elliptic curve decomposes as

$$V_E = \bigoplus_{i=1}^{N} O(p_i - \sigma) \qquad C_1(V) = 0 \Rightarrow \sum_i p_i = 0$$

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- We need another piece of data called **spectral sheaf** or **spectral line bundle**

$$X \times_{B_n} \hat{X} \xrightarrow{\pi_1} X, \qquad X \times_{B_n} \hat{X} \xrightarrow{\pi_2} \hat{X}$$

$$FM^1(V) = R^1\pi_{2*}(\pi_1^* V \otimes \tilde{\mathcal{P}})$$

$$FM^1(V)|_S = \mathcal{L}$$

$$F\hat{M}^0(\mathcal{L}) = \pi_{1*}(\pi_2^* \mathcal{L} \otimes \mathcal{P}) = V$$

• Chern Classes

- Chern classes can be computed by Grothendieck-Riemann-Roch

$$\pi_S : S \to B$$

$$\pi_S^* (\exp(C_1(\mathcal{L}) + C_1(\mathcal{P}_B))Td(S \times_B X)) = Ch(V)Td(X)$$

- If one assume $C_1(\mathcal{L})$ only depends on $\sigma \cap S$ and $\pi_S^* D_b$, the following results can be found: ($[S] = n\,\sigma + \eta$)

$$C_1(\mathcal{L}) = \frac{1}{2}(\pi_S^* C_1(B) - C_1(S)) + \lambda(n\sigma - (\eta - nC_1(B)))$$

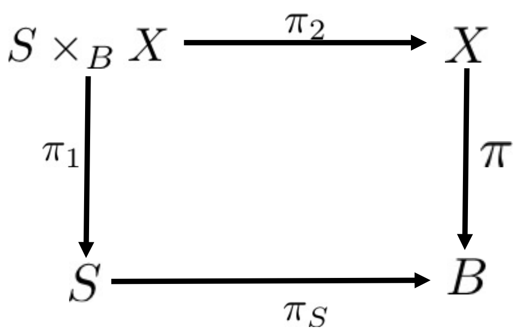
$$C_2(V) = \sigma\eta + \omega$$

$$C_3(V) = 2\lambda\eta(\eta - nC_1(B))$$

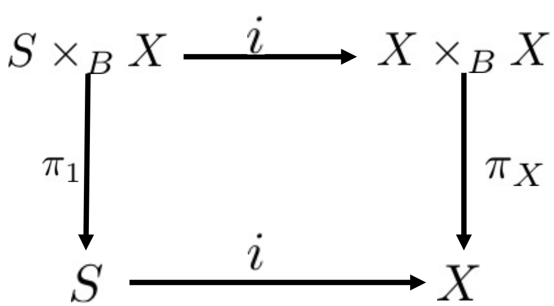
• Matter Content

Hep/th-0405014
Donagi,He,Overut,Reinbacher

- By Lerray, $H^1(X,V) = H^0(B, R^1\pi_* V)$



$$R^1\pi_*(\pi_{2*}(\pi_1^*\mathcal{L} \otimes \mathcal{P})) = \pi_{S*}(R^1\pi_{1*}(\pi_1^*\mathcal{L} \otimes \mathcal{P}))$$
$$= \pi_{S*}(\mathcal{L} \otimes R^1\pi_{1*}\mathcal{P})$$



$$R^1\pi_{1*}(i^*\mathcal{P}) = i^* R^1\pi_{X*}\mathcal{P} = i^* R^1\pi_{X*}\mathcal{O}_{X\times X} = i^*\sigma^* K_B$$

$$H^0(B, R^1\pi_* V) = H^0(\sigma \cap S, \mathcal{L} \otimes K_B)$$

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- If these singularities coincide with the singularities of the CY_3, the blow ups induce different (-2) curves on the double cover.

$$\left(\begin{array}{cccccc|c} 3 & 2 & 1 & 0 & 0 & 0 & 6 \\ 9 & 6 & 0 & 1 & 1 & 1 & 18 \end{array}\right) \longrightarrow \left(\begin{array}{ccccccc|c} 3 & 2 & 1 & 0 & 0 & 0 & 0 & 6 \\ 8 & 5 & 0 & 1 & 1 & 1 & 0 & 16 \\ 9 & 6 & 0 & 0 & 1 & 1 & 1 & 18 \end{array}\right)$$

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- After finding $\tilde{X} \times_B \tilde{X}$ (by blowing up $X \times_B X$) , we can use the GRR theorem to compute the topology of the corresponding SU(2) bundle.
- The result for the special examples $[s] = 2\sigma + 7D$ is,

$$C_1(\mathcal{L}) = \sigma + 5D + \lambda(2\sigma - D) + \sum_{i=1}^{7} \beta_i E_i$$

$$C_2(V) = 7\sigma D + (7\lambda^2 + \sum_{i=1}^{7} \beta_i^2 - 22)D^2 + (\sum_{i=1}^{7} \beta_i^2 - 2\sum_{i=1}^{7} \beta_i + 6)E \cdot D$$

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➢ Holomorphic zero section doesn't intersect with singularities, therefore $\sigma \cdot E_i = 0$ and there is no new contribution in the matter content of EFT.

SU(2) singularity with Rational section

- Consider a case that zero section wraps around a (-2)-curve after blow up.

$$\sigma \cdot E \neq 0$$

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- In this case we have two sections $\sigma_1 = (1,0,0)$ and $\sigma_2 = (-1,1,2)$, the first one is isomorphic to the base. But after blow up it wraps around a (-2)-curve.

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- By repeating the same arguments we get the following result for $[S] = n\,\sigma_1 + k\,D$

$$C_1(\mathcal{L}) = \frac{1}{2}(C_1(B_2) - C_1(S)) + \lambda_1(n\sigma_1 - (k-3n)D + E_\sigma) + \lambda_2(n\sigma_2 - (5n+k)D + (4k(n-1)+n)E_\sigma) + \sum_{i=1}^{k} \beta_i E_i + E_\sigma$$

- $\chi(X,V)$, changes by $2n\,(\lambda_1 + \lambda_2(4k-n) + 1)$.

- $C_2(V)$ and $C_3(V)$ are are complicated functions which depend on the coefficients of E_i and E_σ.

• Conclusion

❖ If the exceptional divisor induce new (-2)-curves, then topology of bundle changes.

❖ Matter content changes only if zero section wraps around some (-2)-curve induced by the exceptional divisor.

❖ The next questions will be

1. How it’s possible to see the change in matter content from the F-theory dual.

2. The contribution of new discrete set of data ($\sum_i \beta_i E_i$) in G_4 flux should be studied.

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Thank you