Quantum Sheaf Cohomology on Grassmannians

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J. Guo, Z. Lu, E. Sharpe, arXiv:1512.08586, 1605.01410

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- OPE $\rightarrow$ Quantum Sheaf Cohomology

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- Some examples (k = 2)

$$
\begin{aligned}
\sigma_{\yng(1)} &= \sigma_1 + \sigma_2, \\
\sigma_{\yng(2)} &= \sigma_1^2 + \sigma_2^2 + \sigma_1\sigma_2, \\
\sigma_{\yng(3)} &= \sigma_1^3 + \sigma_1^2\sigma_2 + \sigma_1\sigma_2^2 + \sigma_2^3, \\
\sigma_{\yng(1,1)} &= \sigma_1\sigma_2, \\
\sigma_{\yng(2,1)} &= \sigma_1^2\sigma_2 + \sigma_1\sigma_2^2, \\
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  $\cong \mathbb{C}\left[\sigma_{(1)}, \sigma_{(2)}, \cdots\right] / \langle D_{k+1}, D_{k+2}, \cdots \rangle$,
  $D_m = \det\left(\sigma_{(1+j-i)}\right)_{1 \le i,j \le m}$

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$R_r \equiv \sum_{i=0}^{\min\{r,n\}} I_i \sigma_{(r-i)} \sigma_{(1)}^i$

Theorem

For a generic deformed tangent bundle $\mathcal{E}$ over $X = G(k,n)$, when $r = n - k + 1$, the kernel of $H^0(\mathrm{Sym}^{\mathrm{r}}(\mathcal{S}^* \otimes \mathcal{S})) \to H^r(\wedge^r \mathcal{E}^*)$ is generated by R_r (relation).

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Theorem [J. Guo, Z. Lu, E. Sharpe, 1605.01410]

For generic deformed tangent bundles, the classical sheaf cohomology ring is the ring of symmetric polynomials in k indeterminates modulo the ideal generated by the R's, which can be given explicitly as

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$$\begin{aligned} &\mathbb{C}[\sigma_{(1)}, \sigma_{(2)}, \cdots]/\langle D_{k+1}, D_{k+2}, \cdots, \sigma_{(n-k+1)}, \sigma_{(n-k+2)}, \cdots \rangle \\ =&\mathbb{C}[\sigma_{(1)}, \cdots \sigma_{(n-k)}]/\langle D_{k+1}, \cdots, \sigma_{(n-1)}, \sigma_{(n)} \rangle \end{aligned}$$

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Quantum Sheaf Cohomology of $G(k,n)$ for Generic (0,2) Deformation [J. Guo, Z. Lu, E. Sharpe, 1512.08586]

$$\mathbb{C}\left[\sigma_{(1)}, \sigma_{(2)}, \cdots\right] / \left\langle D_{k+1}, D_{k+2}, \cdots, R_{(n-k+1)}, \cdots, R_{(n-1)}, R_{(n)} + q, R_{(n+1)} + q\sigma_{(1)}, R_{(n+2)} + q\sigma_{(2)}, \cdots \right\rangle$$

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On the (2,2) locus, this reduces to the ordinary quantum cohomology $\mathbb{C}[\sigma_{(1)}, \cdots \sigma_{(n-k)}]/\langle D_{k+1}, \cdots, \sigma_{(n-1)}, \sigma_{(n)} + (-)^n q\rangle$

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- The OPE ring of the A/2 theory defines the quantum sheaf cohomology.
- Classical sheaf cohomology for generic deformations of Grassmannian is derived by induction.
- Quantum sheaf cohomology of Grassmannian is computed by analyzing the 1-loop effective potential.

Thank You