The quest for precision from the Higgs

[MDG with variously F. Staub, K. Nickel, J. Braathen and P. Slavich, 1411.0675, 1411.4665, 1503.03098, 1511.01904, 1604.05335, 1609.06977, 1706.05372]

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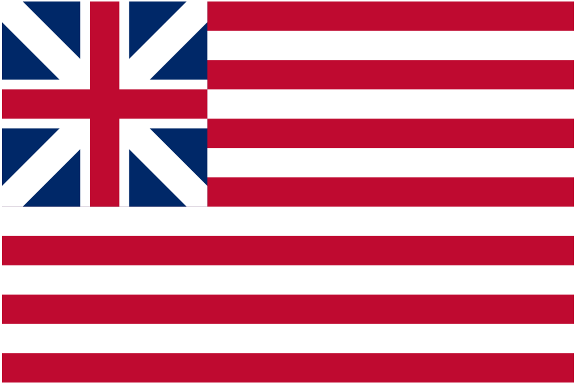
Happy 4th of July!



Happy 5th Birthday to the Higgs boson!

What should we be aiming for?

Finding just the MSSM with TeV-scale SUSY in string theory is hard:

- Exotica
- CMP/GUT requires sequestering in LVS

In the light of null searches, can either look at Split/high-scale SUSY, or alternative models with e.g. better naturalness:

- Still have hints from gauge coupling unification, hints that dark matter might not be just cold, B-physics, $g-2$, etc,
- So maybe keep keeping an open mind (or see e.g. M. Reece's talk) about whether the SUSY scale is low or high.

So can we test from the bottom up whether more general models are allowed? (sadly a retreat from the pre-LHC hope of a large "inverse problem"):

- Yes! This talk is about the study of models beyond the (MS)SM from the low-energy point of view with as much precision as can be desired.
- In particular, answering the question: can they match the measured Higgs mass?
- If we take a high SUSY scale, such models should really not be dismissed out of hand – and may have other benefits (e.g. many vector-like exotics may help the LVS avoid large N_{eff}.)

Overview

- Higher-order corrections to the Higgs mass
- State of the art for generic theories
- A recent challenge: the Goldstone Boson Catastrophe ...
- ... and how to avoid it.

The Higgs mass as a precision electroweak observable

Consider the current experimental accuracy of the Higgs mass measurement:

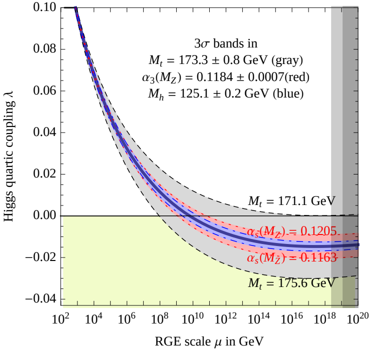
$$\text{ATLAS} + \text{CMS (Moriond 2015)}: \quad m_H = 125.09 \pm 0.21(\text{stat}) \pm 0.11\,(\text{syst.})$$

The uncertainty is tiny!
In the Standard Model:

- The Higgs mass is used to calculate the Higgs quartic coupling $\mathcal{L} \supset -\lambda |H|^2$ (from [Butazzo *et al*, 1307.3536]):

$$\lambda(\mu = m_t) = 0.12604 + 0.00206 \left(\frac{m_h}{\text{GeV}} - 125.15\right)$$

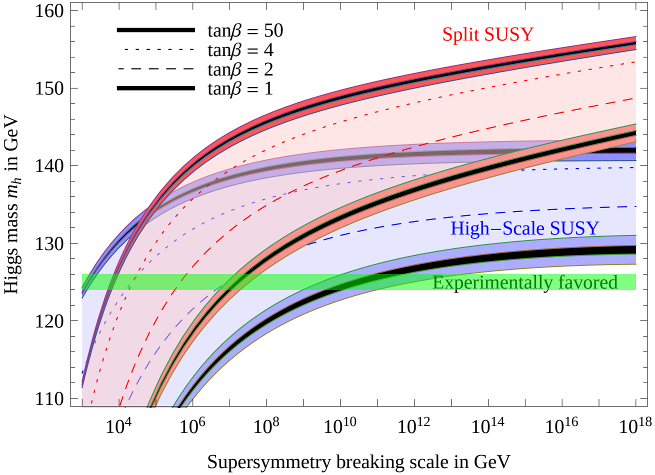
$$-0.00004 \left(\frac{m_t}{\text{GeV}} - 173.34\right) \pm 0.00030$$



- Vital for stability analysis (also needed in principle for future triple/quartic Higgs coupling measurements):
- State-of-the-art computation includes most two-loop effects.

Or for BSM

Taken from [Giudice, Strumia, 1108.6077]:
Predicted range for the Higgs mass



Can now obtain this information for any theory.

Why all the effort?

For many years the standard example has been the MSSM for $\sim$ TeV-scale SUSY:

- Quartic predicted to be determined entirely by gauge couplings at tree level: $\lambda = \frac{1}{8}(g_Y^2 + g_2^2)\cos^2 2\beta$ in heavy M_H limit.
- Hence $\to m_h(\text{tree}) \leqslant M_Z$
- $\delta m_h^2(\text{loops}) \geqslant (86\text{GeV})^2 \gtrsim m_h^2(\text{tree})$
- Can have $\delta m_h(\text{two loops}) \lesssim 10$ GeV
  $\to \delta m_h^2(\text{two loops}) \sim 15\% m_h^2$!
- While at three-loop order, have $\delta m_h \sim$ few hundred MeV,
  $\to \delta m_h^2(\text{three loops}) \lesssim 1\% m_h^2$

Much work has led to: full one-loop calculation, two loops full diagrammatic calculation for $\alpha_s \alpha_t$ only; effective potential approximation and gaugeless limit for (Yukawa coupling)^4 diagrams, and three-loop $\alpha_s^2 \alpha_t$.

Fixed order vs. EFT

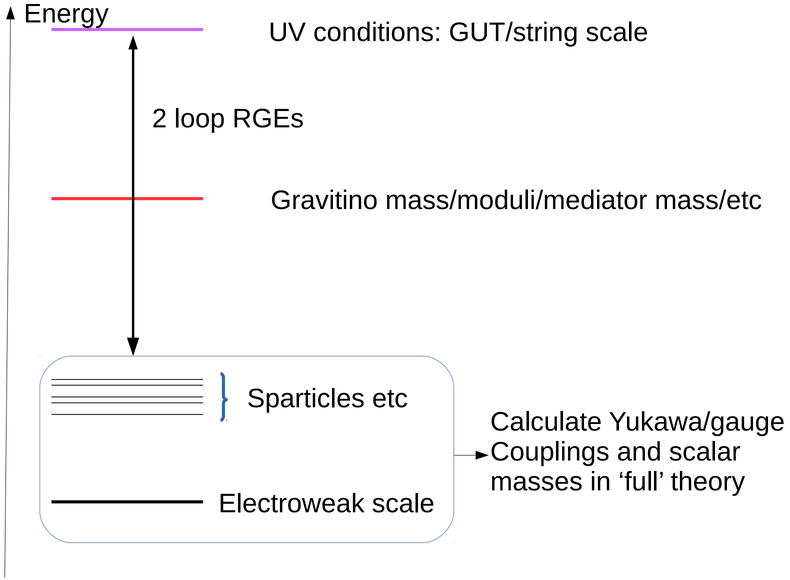
There are also now two approaches to the Higgs mass calculation:

1. Traditional “fixed-order:” include all of the states in the theory and calculate the Higgs mass at e.g. M_{SUSY}. This is still appropriate if there are some light states or Higgs mixing with other scalars.

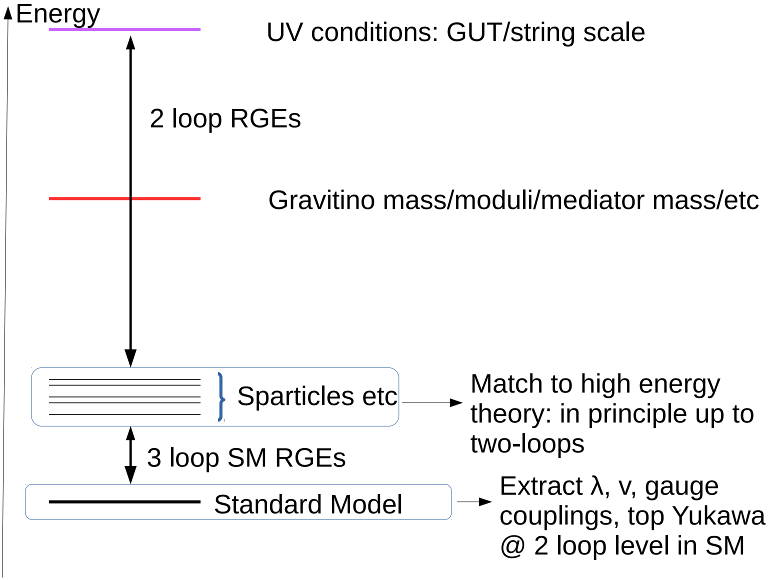
2. Effective Field Theory approach: assume that the Standard Model is valid up to some matching scale. This way large logarithms are automatically resummed through RGEs (3 or more loops in the Standard Model, 2 otherwise) and therefore <u>much</u> more accurate when new physics states are heavy.

The precision available for both is almost identical now, but there are technicalities still to resolve for matching the EFT calculation for a general theory to the SM.

Conventional approach



EFT approach



State of the art for generic models

A summary of what can be done for the generic case:

	Conventional approach	SM	EFT matching
λ/Scalar masses	'Gaugeless EP' 2-loop	'Full' 2-loop/ partial 3 loop	'Gaugeless EP*' 2-loop
Gauge couplings	1-loop	2-loop	1-loop
Yukawas	1-loop	2-loop	1-loop
v	1-loop	2-loop	1-loop
RGEs	2-loop	3 or 4 loops	N/A

- Clearly the extraction of parameters is very important, e.g. in SM $m_h^2 = 2\lambda v^2$ so two-loop extraction of v is technically necessary.
- I will elaborate more in the following on what 'Gaugeless' and 'Gaugeless EP*' mean ...
- But corrections to λ or equivalently computing the scalar masses are the most important – and the subject of this talk!

Extracting λ in the EFT

There is a standard way to calculate threshold corrections to the Higgs quartic:

- Identify combination of scalars in high energy theory (HET) that corresponds to the Higgs: $H = R_{ij}\phi_j$
- Calculate $V_{\text{eff}}(|H|^2)$ in HE theory
- Find $\frac{\partial^4 V_{\text{eff}}}{\partial |H|^4}$.
- At two loops need to include subtleties in corrections to mixing, self energies etc, but for one loop this works fine.
- Main problem is that generic expressions are painful and not known beyond one loop.

Extracting λ in the EFT: alternative approach

The alternative method, implemented in `FlexibleSUSY` and `SARAH` is:

- Calculate $M_h^2(m_h^2)$ (i.e. pole mass) in SM and in HET at the matching scale M:

$$M_{h,SM}^2(p^2) = 2\lambda v^2 + \Delta M_{h,SM}^2(p^2)$$

- Set them equal:

$$\rightarrow \lambda = \frac{1}{2v^2}\left[M_{h,HET}^2(m_h^2) - \Delta M_{h,SM}^2(m_h^2)\right]$$

Here we only compute two-point diagrams $\rightarrow$ computationally much easier.

- Hence a code (`SARAH`) that can compute the Higgs mass via the conventional method can also calculate the λ thresholds ...
- However: there are subtleties involving subleading logs $\rightarrow$ for general theories the results available are not genuinely two-loop $\rightarrow$ work in progress.

Calculation of the Higgs mass

The Higgs mass is corrected order by order through two effects:

1. Self energy corrections

$$m^2_{\text{pole}} = m^2_0 + \Pi(m^2_{\text{pole}})$$

2. Shifts to the minimum conditions: we define the potential in terms of real scalars with vevs v to be

$$\begin{aligned} V(v) =& V^{\text{tree}} + \Delta V \equiv \frac{1}{2} m^2_{\text{run}} v^2 + V_\lambda^{\text{tree}} + \Delta V \\ \to 0 =& m^2_{\text{run}} v + \left(\frac{\partial V_\lambda^{\text{tree}}}{\partial v} + \frac{\partial \Delta V}{\partial v} \right) \end{aligned}$$

If we take v as fixed to all orders (which is convenient since couplings depend on v) we must shift m^2_{run} so that

$$\to m^2_0 = \frac{\partial^2 V^{\text{tree}}}{\partial v^2} = m^2_{\text{run}} + \frac{\partial^2 V_\lambda^{\text{tree}}}{\partial v^2} = \underbrace{\left(\frac{\partial^2 V_\lambda^{\text{tree}}}{\partial v^2} - \frac{1}{v} \frac{\partial V_\lambda^{\text{tree}}}{\partial v} \right)}_{\text{tree}-\text{level mass}} - \frac{1}{v} \frac{\partial \Delta V}{\partial v}.$$

So we need the tadpole diagrams as well as self-energies to calculate the mass; note that if we took the masses fixed instead of the vevs we would still have a shift in the mass due to a shift in v (c.f. Higgs tree-level mass of $2\lambda v^2$).

The effective potential approach

Now we turn to calculating two-loop corrections to the Higgs mass/quartic.
One significant simplification to calculations is to take $p^2 = 0$; this is then equivalent to taking

$$\Pi(0) = \frac{\partial^2 \Delta V}{\partial v^2}.$$

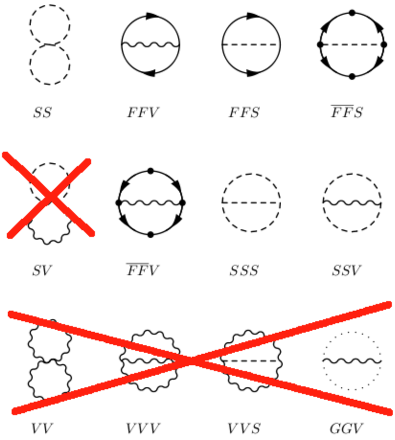
Hence the "effective potential limit." When the scale of new physics is above the electroweak scale this is a good approximation, and is better than might be expected even for the Standard Model.

The gaugeless limit

The 'gaugeless limit' is a popular simlification in both SM and BSM:

- Set $g_Y = g_2 = 0$ in two-loop calculation (and any other couplings of broken gauge groups in BSM models) – but keep the important g_3!

- Justified by smallness of α: even if g_2 is not very small, $\alpha_2 \equiv \alpha / s_W^2 \simeq 0.03$, c.f. $\alpha_t \simeq 0.08$, $\alpha_s \simeq 0.12$
- ... and also by lack of large logs involving weak bosons. The approximation works very well – typical correction to the Higgs mass of $\mathcal{O}(10 - 100)$ MeV.
- On the other hand, it dramatically simplifies calculations.
- Has a special place in the MSSM because $\lambda \propto g_Y^2 + g_2^2$ at tree level $\rightarrow$ kills Higgs self-couplings in the loops.



Generic calculations

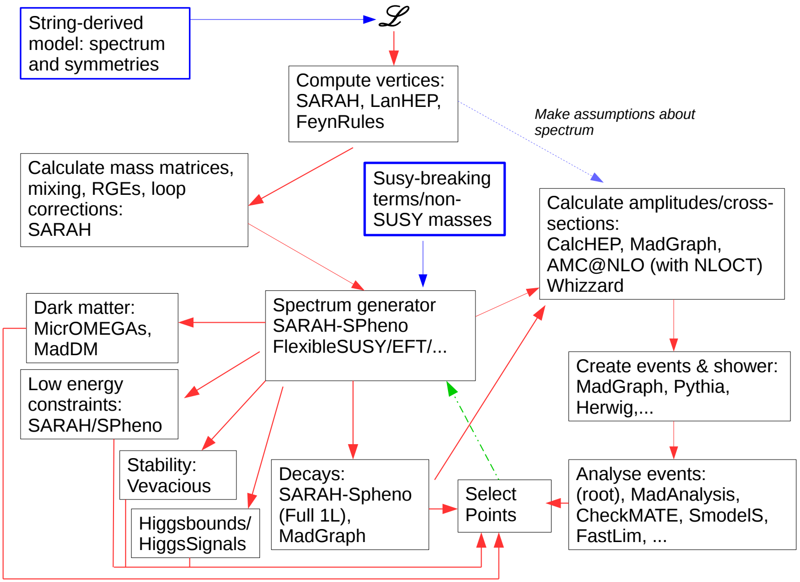
- Some contributions of the effective potential are known for the Standard Model up to three and four loop order ...
- Otherwise it is only known in Landau gauge up to two loops. [S. Martin, 01] gave the expression in dimensional regularisation ($\overline{\mathrm{DR}}$ and $\overline{\mathrm{MS}}$) for generic theories.
- [Martin, '03] gave the two-loop scalar self-energies up to $\mathcal{O}(g^2)$ in gauge couplings (don't need g^4 in the gaugeless limit).
- In [MDG, Nickel, Staub 1503.03098] we calculated the tadpoles, and substantial simplifications for massless gauge fields.
- We have implemented in `SARAH` a diagrammatic calculation for self-energies and tadpoles in a "generalised effective potential and gaugeless limit."

SARAH: a tool for BSM model builders

So what is `SARAH` ?

- `Mathematica` package created by F. Staub, with now many contributions from MDG.
- Takes an input model file for any SUSY or non-SUSY model.
- Specify: gauge groups, matter content, superpotential/couplings in Lagrangian.
- Relevant for this talk: spectrum generation with `SPheno`. Produces fortran code which compiles against the `SPheno` library to generate spectrum and precision observables etc for the model.
- Will calculate two-loop RGEs, one-loop masses for all particles in $\overline{\mathrm{DR}}'$ (SUSY) or $\overline{MS}$ (non-SUSY) models, one-loop decays, ...
- Can specify input parameters at any scale: TeV, ~~SUSY~~ scale, GUT scale ...

A web of codes from the top down



The Goldstone Boson Catastrophe

But there is a technical barrier for any theory other than the gaugeless limit of the MSSM: the Goldstone Boson Catastrophe. Note that this includes the Standard Model where it was studied by [Martin, '14], [Elias-Miro, Espinosa, Konstandin, '14]!

- Consider for simplicity the Abelian Goldstone Model of one complex scalar $\Phi = \frac{1}{\sqrt{2}}(v + h + iG)$ and tree-level potential

$$V = \mu^2|\Phi|^2 + \lambda|\Phi|^4.$$

- At tree level, the tadpole equation gives $\mu^2 + \lambda v^2 = 0$, and the masses are $m_G^2 = \mu^2 + \lambda v^2, M_h^2 = \mu^2 + 3\lambda v^2$.
- But we use $m_G^2 \equiv \mu^2 + \lambda v^2$ to calculate loops, and once we include loop corrections we have

$$0 = \mu^2 + \lambda v^2 + \frac{\partial \Delta V}{\partial v}$$

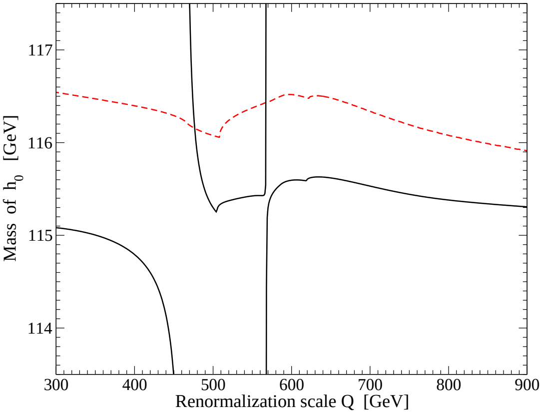
- ... hence $m_G^2 = \mathcal{O}(1 - \text{loop})$ and is of indefinite sign!
- In fact, at two loops we find (with $A(x) \equiv x(\log x/Q^2 - 1)$)

$$0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\Big[3A(m_h^2) + A(m_G^2)\Big]}_{\text{1-loop}}$$

$$\underbrace{+ \frac{\log \frac{m_G^2}{Q^2}}{(16^2)^2}\Big[3\lambda^2 v A(m_G^2) + \frac{4\lambda^3 v^3}{M_h^2} A(M_h^2)\Big] + \overbrace{\cdots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}}$$

GB Catastrophe in the MSSM

The problem was identified early on when trying to use the effective potential approach on the full MSSM potential – From S. Martin [hep-ph/0211366]:



Solid line: including EW effects, dashed line without

This shows both the GB catastrophe near $Q = 568$ GeV and the ‘Higgs boson catastrophe’ near 463 GeV.

Ways out

When we implemented the two-loop calculation in `SARAH`, we had to confront the problem. What to do?

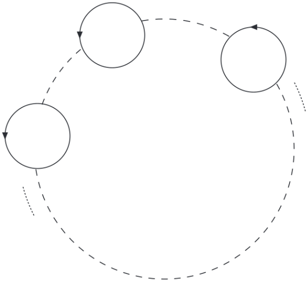
- Calculations in the Standard Model have used Feynman gauge. In general this is much more complicated, the generic expressions are not available – and it anyway does not actually completely solve the problem!
- Otherwise we could simply ignore the phases introduced in the potential and try to find a renormalisation scale Q where $m_G^2 > 0$. But the masses will have a huge sensitivity to the IR effects so we can no longer trust the calculation, because of the way that spectrum generators implement the corrections.
- Indeed, the problem is more serious, because all loop computations were performed using the tree-level masses, which would give <u>exactly</u> massless Goldstones in Landau gauge $\to$ everywhere divergent tadpoles and self-energies.

Resummation

A solution for the Standard Model was proposed in [Martin, '14], [Elias-Miro, Espinosa, Konstandin, '14]:

The daisy diagram contributes the most singular term at any fixed loop order; it has the most soft Goldstone propagators – each term looks like

$$\int d^4 q \frac{(\Pi_{GG}(q^2))^n}{(q^2 - m_G^2)^n} \sim (\Pi_{GG}(0))^n \frac{\partial^n f(m_G^2)}{\partial (m_G^2)^n}$$



- $f(x) \equiv \frac{1}{4} x^2 (\overline{\log}\, x - \frac{3}{2})$ so $f(0) = 0$, whereas $f(m_G^2)$ contains a logarithmic term.
- But if we sum it to all orders, then we will just find $f(m_G^2 + \Pi_{GG}(0))$ – which is zero and has finite first derivatives!

So we should use instead use the resummed potential

$$\hat{V}_{\text{eff}} \equiv V_{\text{eff}} + \frac{1}{16\pi^2} \left[f(m_G^2 + \Delta) - \sum_{n=0}^{l-1} \frac{\Delta^n}{n!} \left(\frac{\partial}{\partial m_G^2} \right)^n f(m_G^2) \right].$$

The two potentials only differ by terms of order $l+1$. The two papers then differ in how to define Δ (it is not quite $\Pi_{GG}(0)$)

Generalising

If we want to apply this to general theories, however, we have two problems:

1. Identifying the Goldstone boson(s) among the scalars: in general the fields can mix!
2. Taking derivatives of the potential as a function of masses and couplings generally means taking derivatives of mixing matrices.

[Martin, Kumar '16] applied this to the MSSM with CP conservation, where they could use 2×2 matrices and do all the derivatives explicitly.

We can do better by taking all of the derivatives <u>implicitly</u>.

We can do better still by adopting a different solution.

On-shell scheme

We saw that we can cure the IR divergences by resumming the Goldstone boson propagators, so that the effective mass in the loop functions became $m_G^2 + \Delta = 0$. But we can do this more directly by just putting the Goldstone boson on shell:

$$(m_G^2)^{\text{run.}} \equiv (m_G^2)^{\text{OS}} - \Pi_{GG}((m_G^2)^{\text{OS}})$$

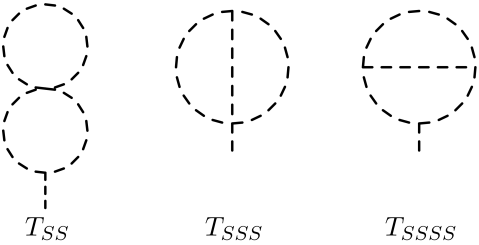
We can do this directly in the tadpole equations – and also the self-energies! So then there should be no need to take derivatives of couplings ... exactly what we want. For example, applying the above shift to the one loop tadpole gives a two-loop correction:

$$\frac{\partial V}{\partial v} \supset \frac{\lambda v}{16\pi^2} A(m_G^2) = \frac{\lambda v}{16\pi^2}\left[\underbrace{A((m_G^2)^{\text{OS}}}_{\to 0} - \underbrace{\Pi_{GG}((m_G^2)^{\text{OS}}) \log \frac{(m_G^2)^{\text{OS}}}{Q^2}}_{\text{cancels divergent part}} + \underbrace{\ldots}_{3-\text{loop}}\right]$$

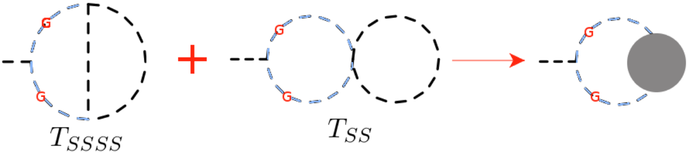
We also see that $\Pi_{GG}((m_G^2)^{\text{OS}}) = \Pi_g(0)$ (at least at this loop order) automatically!

Illustration

To see why this works, let us look at the scalar-only case. There are three classes of tadpole diagrams:

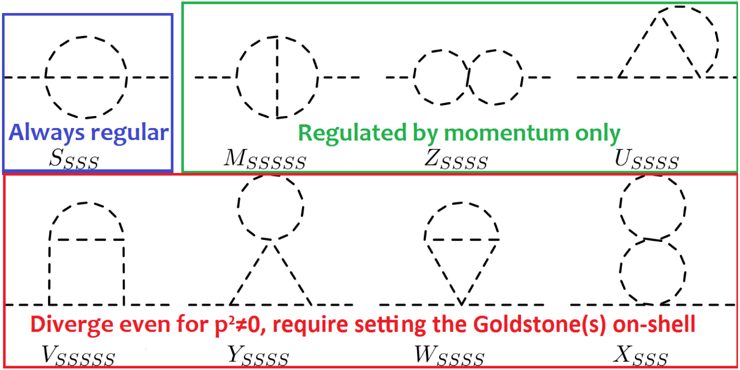


We find that the divergences only come from the T_{SS} and T_{SSSS} topologies, and they correspond to a Goldstone self-energy as a subdiagram and exactly cancel out against the on-shell shift:



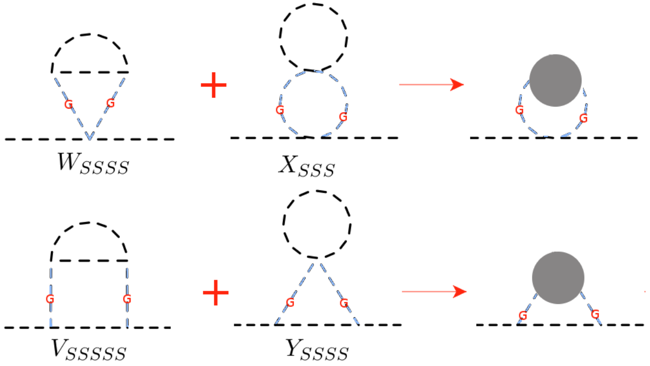
Mass diagrams

We also find that we can apply our on-shell scheme to the cancellation of divergences in self-energies! This seemed hopeless in the former approaches ... We can divide the topologies into three categories:



Mass diagram divergences

Again we find that the divergences in m_G^2 arise from Goldstone boson propagator subdiagrams:



... and once more the one-loop shifts from our on-shell scheme exactly cancel the divergences (but leave a finite momentum-dependent piece).

Generalised effective potential limit

Since we see that there are classes of diagrams that are divergent when the $p^2 \equiv s \neq 0$ and the Goldstone bosons are on-shell, the obvious response is that we cannot avoid using momentum dependence – but this is computationally expensive. Instead, we can expand the self-energies as:

$$\Pi^{(2)}_{ij}(s) = \frac{\overline{\log}(-s)}{s}\Pi^{(2)}_{-1\,l,ij} + \frac{1}{s}\Pi^{(2)}_{-1,ij} + \Pi^{(2)}_{l^2,ij}\,\overline{\log}^2(-s) + \Pi^{(2)}_{l,ij}\,\overline{\log}(-s) + \Pi^{(2)}_{0,ij} + \sum_{k=1}^{\infty}\Pi^{(2)}_{k,ij}\frac{s^k}{k!}$$

If we discard all terms $\mathcal{O}(s)$ and higher, we have a <u>generalised effective potential approximation</u>! We can find closed forms for the singular terms, e.g.

$$U(0,0,0,u) = (\overline{\log}\,u - 1)\,\overline{\log}(-s) - \frac{\pi^2}{6} + \frac{5}{2} - 2\,\overline{\log}\,u - \frac{1}{2}\overline{\log}^2 u + \mathcal{O}(s).$$

This turns out to be a very good approximation, e.g. even in the Standard Model:

	SARAH/SPheno		SMH (code by Martin & Robertson)	
ξ	1	0.01	0	
2ℓ momentum dependence	partial $s = m_h^2\|^{\text{tree}}$	partial $s = m_h^2\|^{\text{tree}}$	full iterative	none $s = 0$
$m_h^{2\ell}$ (GeV)	125.083	125.134	125.176	125.121

Conclusions

- It is now possible to study the phenomenology of any renormalisable BSM theory with high precision
- For example, `SARAH` now gives the most precise value of the Higgs mass in the NMSSM, and even for the MSSM with CPV is the only code appropriate for top-down analysis ($\overline{\mathrm{DR}}'$ as opposed to on-shell scheme inputs).
- From the top-down perspective, can potentially rule models out based on field content, choices of couplings and mass scales.
- Ongoing work will further refine the precision: EFT approach, including EW contributions, ...
- I haven't talked about Higgs couplings, decays etc but these can also be studied.