Heterotic Target Space Duality and F-theory

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Based on work done by:

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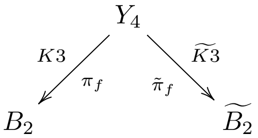
String Pheno - July 6, 2017

- Target Space Duality (heterotic): $(X, V) \sim (\widetilde{X}, \widetilde{V})$
- Het/F-theory Duality:
  - Heterotic: $\pi_h : X_n \xrightarrow{\mathbb{E}} B_{n-1}$ elliptic fibration
  - F-theory: $\pi_f : Y_{n+1} \xrightarrow{K3} B_{n-1}$ where $\rho_f : Y_{n+1} \xrightarrow{\mathbb{E}} B_n$, $\sigma_f : B_n \xrightarrow{\mathbb{P}^1} B_{n-1}$
- Question (conjectured by Blumenhagen): if in Target Space Duality, $\pi_h : X_3 \xrightarrow{\mathbb{E}} B_2$ and $\tilde{\pi}_h : \widetilde{X_3} \xrightarrow{\mathbb{E}} \widetilde{B_2}$ are both elliptically fibered, is there a Y_4 in F-theory s.t.



Review of Target Space Duality (Distler, Kachru, Blumenhagen)

- Abelian, massive 2D theory $\rightarrow$ $(0,2)$ GLSM
- Multiple $U(1)$ gauge fields $A^{(\alpha)}$ with $\alpha = 1, ..., r$
- Chiral and Fermi superfields can be written in a charge matrix:

$$\begin{array}{cccc||cccc} \multicolumn{4}{c||}{x_i} & \multicolumn{4}{c}{\Gamma^j} \\ \hline Q_1^{(1)} & Q_2^{(1)} & \dots & Q_d^{(1)} & -S_1^{(1)} & -S_2^{(1)} & \dots & S_c^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Q_1^{(r)} & Q_2^{(r)} & \dots & Q_d^{(r)} & -S_1^{(r)} & -S_2^{(r)} & \dots & S_c^{(r)} \end{array}$$
$$\begin{array}{cccc||cccc} \multicolumn{4}{c||}{\Lambda^a} & \multicolumn{4}{c}{p_l} \\ \hline N_1^{(1)} & N_2^{(1)} & \dots & N_\delta^{(1)} & -M_1^{(1)} & -M_2^{(1)} & \dots & -M_\gamma^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ N_1^{(r)} & N_2^{(r)} & \dots & N_\delta^{(r)} & -M_1^{(r)} & -M_2^{(r)} & \dots & -M_\gamma^{(r)} \end{array} \quad (1)$$

GLSM is defined via a superpotential:

$$S = \int d^2 z d\theta \left[\sum_j \Gamma^j G_j(x_i) + \sum_{l,a} P_l \Lambda^a F_a^l(x_i) \right] \quad (2)$$

G_j and F_a^l are quasi-homogeneous polynomials with degrees S_j and $M_a - N_l$

F-term and D-term potential:

$$V_F = \sum_j \left|G_j(x_i)\right|^2 + \sum_a \left|\sum_l p_l F_a^l(x_i)\right|^2 \tag{3}$$

$$V_D = \sum_{\alpha=1}^{r} \left(\sum_{i=1}^{d} Q_i^{(\alpha)} |x_i|^2 - \sum_{l=1}^{\gamma} M_l^{(\alpha)} |p_l|^2 - \xi^{(\alpha)} \right)^2 \tag{4}$$

Fayet-Iliopoulos (FI) parameter ξ controls the phase:

- "geometric" phase: Calabi-Yau manifold/Vector bundle
- "nongeometric" phase: Landau-Ginzburg orbifold
- hybrid phase

For Landau-Ginzburg orbifold with a superpotential:

$$\mathcal{W}(x_i, \Lambda^a, \Gamma^i) = \sum_j \Gamma^j G_j(x_i) + \sum_{l,a} P_l \Lambda^a F_a^l(x_i) \tag{5}$$

Observation: An exchange/relabeling of the functions G_j and F_a will not affect the Landau-Ginzburg model.

Procedure:

- Start from geometric phase (X, V)
- Find a nongeometric phase, rescale the fields
- Go back to geometric phase and get $(\widetilde{X}, \widetilde{V})$

Example 0

Consider the following configuration:

x_i							Γ^j		Λ^a				p_l
0	0	0	1	1	1	1	−2	−2	1	0	0	2	−3
1	1	1	2	2	2	0	−4	−5	0	1	1	6	−8

(6)

Here for (X,V): $||G_1|| = (2,4)$, $||G_2|| = (2,5)$, $||F_3^1|| = (3,7)$, $||F_4^1|| = (1,2)$.
Rescale to $(\widetilde{X}, \widetilde{V})$: $||\tilde{G}_1|| = (3,7)$, $||\tilde{G}_2|| = (1,2)$, $||\tilde{F}_3^1|| = (2,4)$, $||F_4^1|| = (2,5)$

x_i							Γ^j		Λ^a				p_l
0	0	0	1	1	1	1	−3	−1	1	0	1	1	−3
1	1	1	2	2	2	0	−7	−2	0	1	4	3	−8

(7)

Degree of freedom count:

$$h^*(V) = (0,120,0,0) \quad h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 2 + 68 + 322 = 392$$

$$h^*(\widetilde{V}) = (0,120,0,0) \quad h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(End_0(\widetilde{V})) = 2 + 95 + 295 = 392$$

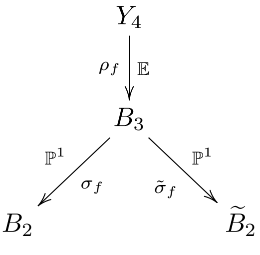
Landscape scan by Blumenhagen + Rahn, agreement in nearly all ∼80,000 examples.

Heterotic side: $\pi_h : X_3 \xrightarrow{\mathbb{E}} B_2$ elliptic fibration

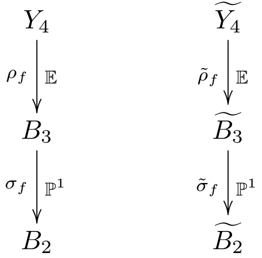
F-theory side: $\pi_f : Y_4 \xrightarrow{K3} B_2$ where $\rho_f : Y_4 \xrightarrow{\mathbb{E}} B_3$, $\sigma_f : B_3 \xrightarrow{\mathbb{P}^1} B_2$

Question: if $\pi_h : X_3 \xrightarrow{\mathbb{E}} B_2$ and $\tilde{\pi}_h : \widetilde{X_3} \xrightarrow{\mathbb{E}} \widetilde{B_2}$ are both elliptically fibered, is there a CY 4-fold, Y_4 in F-theory with multiple fibrations leading to dual 4D theories, which is Target Space Dual

Possibility 1:



Possibility 2:



Example 1

Start from the following (X,V)

x_i	Γ^j		Λ^a									p_l		
$\mathbb{P}^1$	-1	-1	1	1	0	0	0	0	0	0	1	-1	-1	-1
$\mathbb{P}^2$	-1	-2	0	0	1	1	1	0	0	0	2	-1	-2	-2
$\mathbb{P}^2$	-1	-2	0	0	0	0	0	1	1	1	1	-1	-2	-1

(8)

Intermediate step: add a $\mathbb{P}^1$ to resolve singularity

x_i	Γ^j			Λ^a									p_l		
$\mathbb{P}^1$	0	-1	-1	1	1	0	0	0	0	0	0	1	-1	-1	-1
$\mathbb{P}^2$	0	-1	-2	0	0	1	1	1	0	0	0	2	-1	-2	-2
$\mathbb{P}^2$	0	-1	-2	0	0	0	0	0	1	1	1	1	-1	-2	-1
$\mathbb{P}^1$	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0

(9)

Rescale to get $(\widetilde{X}, \widetilde{V})$

x_i	Γ^j			Λ^a									p_l		
$\mathbb{P}^1$	0	-1	-1	1	1	0	0	0	0	0	0	1	-1	-1	-1
$\mathbb{P}^2$	0	-1	-2	0	0	1	1	1	0	0	0	2	-1	-2	-2
$\mathbb{P}^2$	-1	-1	-1	0	0	0	0	0	1	1	0	2	-1	-2	-1
$\mathbb{P}^1$	-1	0	-1	0	0	0	0	0	0	0	1	0	0	-1	0

(10)

Here $X = \begin{array}{c} \mathbb{P}^1 \\ \mathbb{P}^2 \\ \mathbb{P}^2 \end{array} \left[\begin{array}{cc} 1 & 1 \\ 1 & 2 \\ \hline 1 & 2 \end{array} \right]$ $\quad \widetilde{X} = \begin{array}{c} \mathbb{P}^1 \\ \mathbb{P}^2 \\ \mathbb{P}^2 \\ \mathbb{P}^1 \end{array} \left[\begin{array}{c|cc} 0 & 1 & 1 \\ 0 & 1 & 2 \\ \hline 1 & 1 & 1 \\ 1 & 0 & 1 \end{array} \right]$

elliptically fibered on $B_2 = \mathbb{P}^2$ $\quad \widetilde{B_2} = \begin{array}{c} \mathbb{P}^2 \\ \mathbb{P}^1 \end{array} \left[\begin{array}{c} 1 \\ 1 \end{array} \right]$ respectively

with zero-sections

$$\sigma_1(X) = (-J_1 + J_2 + J_3), \quad \sigma_2(X) = (2J_1 - J_2 + 5J_3)$$

$$\sigma_1(\widetilde{X}) = (-J_1 + J_2 + J_3), \quad \sigma_2(\widetilde{X}) = (2J_1 - J_2 + 4J_3 + J_4)$$

Degree of freedom count:

$$h^*(V) = (0, 57, 0, 0) \quad h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 3 + 60 + 292 = 355$$

$$h^*(\widetilde{V}) = (0, 57, 0, 0) \quad h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(End_0(\widetilde{V})) = 4 + 53 + 298 = 355$$

Spectrum count:

$$\mathbf{15}'s: \quad h^1(\wedge^2 V) = 115 \quad h^1(\wedge^2 \widetilde{V}) = 115$$

$$\mathbf{20}'s: \quad h^1(\wedge^3 V) = 2 \quad h^1(\wedge^3 \widetilde{V}) = 2$$

$$\overline{\mathbf{15}}'s: \quad h^1(\wedge^4 V) = 1 \quad h^1(\wedge^4 \widetilde{V}) = 1$$

Since $h^{1,1}(B_2) = 1 \neq h^{1,1}(\widetilde{B_2}) = 2$, is the conjecture true?

Example 2 (non-trivial rewriting)

Consider $(X, V) =$

x_i							Γ^j		Λ^a							p_l	
3	2	1	0	0	0	0	−6	0	3	2	1	0	0	0	0	−6	0
0	0	−2	1	1	1	1	0	−2	0	0	−2	1	1	1	1	0	−2

(11)

$(\widetilde{X}, \widetilde{V}) =$

x_i									Γ^j			Λ^a							p_l	
3	2	1	0	0	0	0	0	0	−6	0	0	3	2	1	0	0	0	0	−6	0
0	0	−2	1	1	1	1	0	0	0	−1	−1	0	0	−2	1	1	0	2	0	−2
0	0	0	0	0	0	0	1	1	0	−1	−1	0	0	0	0	0	1	0	0	−1

(12)

$\widetilde{X}$ is a non-trivial rewriting of X. Conjecture is true in this case.

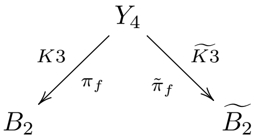
Degree of freedom count:

$$h^*(V) = (0, 241, 1, 0) \quad h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 3 + 243 + 1074 = 1320$$

$$h^*(\widetilde{V}) = (0, 241, 1, 0) \quad h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(End_0(\widetilde{V})) = 3 + 243 + 1074 = 1320$$

Conclusion and Future Work

- Understand Target Space Duality in F-theory: if there is a Y_4 in F-theory with two fibrations leading to dual 4-dimensional theories, are they related by Target Space Duality
- For our non-trivial rewriting case, this conjecture is true



- Future work: compare degree of freedom and spectrum for existing examples, find Y_4
- Study more examples, or prove/disprove the conjecture

Thank you!

Appendix slides

A possible Y_4 for example 1

$$\mathrm{X} = \begin{array}{c} \mathbb{P}^1 \\ \mathbb{P}^2 \\ \mathbb{P}^2 \end{array} \left[\begin{array}{cc} 1 & 1 \\ 1 & 2 \\ \hline 1 & 2 \end{array} \right] \quad \text{fibered on } B_2 = \mathbb{P}^2$$

$$\widetilde{X} = \begin{array}{c} \mathbb{P}^1 \\ \mathbb{P}^2 \\ \mathbb{P}^2 \\ \mathbb{P}^1 \end{array} \left[\begin{array}{c|cc} 0 & 1 & 1 \\ 0 & 1 & 2 \\ \hline 1 & 1 & 1 \\ 1 & 0 & 1 \end{array} \right] \quad \text{fibered on } \widetilde{B_2} = \begin{array}{c} \mathbb{P}^2 \\ \mathbb{P}^1 \end{array} \left[\begin{array}{c} 1 \\ 1 \end{array} \right]$$

$$\text{A possible } Y_4 \text{ could be: } \begin{array}{c} \mathbb{P}^2 \\ \mathbb{P}^1 \\ \mathbb{P}^2 \\ \mathbb{P}^1 \\ \mathbb{P}^2 \end{array} \left[\begin{array}{cc|cc} 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ \hline 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{array} \right] \quad \text{with } B_3 = \begin{array}{c} \mathbb{P}^2 \\ \mathbb{P}^1 \\ \mathbb{P}^2 \end{array} \left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{array} \right]$$

This B_3 has two $\mathbb{P}^1$ fibrations, with $B_2 = \mathbb{P}^2$, and $\widetilde{B_2} = \begin{array}{c} \mathbb{P}^1 \\ \mathbb{P}^2 \end{array} \left[\begin{array}{c} 1 \\ 1 \end{array} \right]$

Example 3

Start from this one

x_i							Γ^j	Λ^a					p_l	
3	2	1	0	0	0	0	−6	1	0	1	1	1	−1	−3
0	0	−2	1	1	0	0	0	1	3	−2	2	1	−1	−4
0	0	−2	0	0	1	1	0	0	3	−2	3	0	−1	−3

(13)

Add repeated entries:

x_i									Γ^j			Λ^a					p_l	
3	2	1	0	0	0	0	**1**	**0**	−6	**−1**	**0**	1	0	1	1	1	−1	−3
0	0	−2	1	1	0	0	**−3**	**1**	0	**3**	**−1**	1	3	−2	2	1	−1	−4
0	0	−2	0	0	1	1	**−2**	**1**	0	**2**	**−1**	0	3	−2	3	0	−1	−3

(14)

Perform interchange and cancel out repeated entries

x_i							Γ^j	Λ^a					p_l	
3	2	1	0	0	0	0	−6	1	0	1	1	1	−1	−3
0	0	−3	1	1	1	0	0	0	4	−2	2	1	−1	−4
0	0	−2	0	0	1	1	0	0	3	−2	3	0	−1	−3

(15)

Spectrum do not match. Possible reasons: bundle stability; existence of LG phase (checked exist); singularity (checked smooth); mass terms (checked no); other