Holography for Composite Inflation

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Cosmological Inflation:

Standard description:

- expansion driven by the potential energy of a scalar field $\varphi$ called *inflaton*

- weakly coupled Lagrangian for the inflaton within QFT framework:

$$S = \int d^4x \sqrt{-\text{det} g} \left[ \frac{R}{2} + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right]$$

- preferably: small field models

  $$(\Delta \varphi \ll M_P \Rightarrow \text{EFT reliable})$$
BUT:

\[ \eta \text{ problem:} \]

Recall the slow roll conditions:

\[ \varepsilon = \frac{V'(\varphi)}{V(\varphi)} \ll 1, \quad \eta = \frac{V''(\varphi)}{V(\varphi)} \ll 1 \]

(consistency with observations \(\Rightarrow\) slow roll inflation)

However: Quantum corrections drive inflaton mass \(m^2_\varphi = V''\) to cutoff of effective theory (at least Hubble scale \(H \approx \sqrt{V}\))

\[ \rightarrow \Delta \eta \approx O(1) \text{ or larger } \Rightarrow \text{ inflation ends prematurely} \]

Hence need a symmetry... (ex.: axion monodromy inflation...)
Composite Inflation:

A possible different approach:

Inflaton - a composite state in a strongly coupled gauge theory

→ inflaton mass dynamically fixed ⇒ no $\eta$ problem!

Recently was argued that tensor-to-scalar ratio $r$ can be large in such models [P. Channuie, K. Karwan, arXiv:1404.5879]

Our aim: Use Gauge/Gravity Duality (GGD) to study this class of inflationary models
Gauge/gravity duality:

D-branes: open string BCs $\leftrightarrow$ SUGRA solutions

strong coupling $\leftrightarrow$ weak coupling

$\Rightarrow$ Can use classical supergravity to learn about strongly coupled gauge theories

Some potential applications:

- high $T_c$ superconductivity
- quark-gluon plasma [viscosity/entropy density]
- dynamical electroweak symmetry breaking
- composite (in particular, glueball) inflation
Gravity Backgrounds

Solutions of 10d SUGRA equations of motion

**Consistent truncation: significant simplification**

(Recall: Consistent truncation means that every solution of the lower dimensional action lifts to a solution of the full 10d action)

We will investigate a 5d consistent truncation of type IIB, established in [M. Berg, M. Haack, W. Muck, hep-th/0507285]

(This encompasses MN, KS solutions, but we will look for nonsusy ones.)

→ 5d fields:

- metric: \( g_{IJ}(x^I) \)
- 6 scalars: \( \phi(x^I), p(x^I), q(x^I), u(x^I), v(x^I), b(x^I) \)
5d action:

Let us denote \( \{ \phi^i \} = \{ \phi, p, q, u, v, b \} \):

\[
S = \int d^5x \sqrt{-\det g} \left[ -\frac{R}{4} + \frac{1}{2} G_{ij}(\varphi) \partial_I \varphi^i \partial_I \varphi^j + V(\varphi) \right],
\]

\( G_{ij}(\varphi) \) - sigma model metric ,

\( V(\varphi) \) - complicated potential

Equations of motion:

\[
\nabla^2_{5d} \varphi^i + G^{i}_{jk} g^{IJ}(\partial_I \varphi^j)(\partial_J \varphi^k) - V^i = 0 ,
\]

\[
-R_{IJ} + 2G_{ij}(\partial_I \varphi^i)(\partial_J \varphi^j) + \frac{4}{3} g_{IJ} V = 0 ,
\]

\( G^{i}_{jk} \) - Christoffel symbols for \( G_{ij} \), \( V^i = G^{ij} \partial_{\varphi^j} V \).
dS and Inflationary Solutions

Want to find a solution with the 5d metric:

\[ ds_{5d}^2 = e^{2A(r)} \left[ -dt^2 + a(t)^2 d\vec{x}^2 \right] + dr^2 \]

[K. Ghoroku, M. Ishihara, A. Nakamura, hep-th/0609152: Used a 10d solution in IIB with such external 5d metric and \( a(t) = e^{\sqrt{\frac{A}{3}} t} \) to study gauge theory in dS space. But the two scalars in that solution: \( \phi(r), C(r) \Rightarrow \text{not compatible with above consistent truncation.} \]

Hubble parameter:

\[ H(t) \equiv \frac{\dot{a}(t)}{a(t)} \quad \left( \Rightarrow \dot{H} = \frac{\ddot{a}}{a} - H^2 \right) \]

Note:

- dS space: \( H = \text{const} \)

- Slow roll inflation: \( H = H(t), \) but \( \dot{H} \) small

[More precisely: \( \ddot{a} > 0 \iff \epsilon \equiv -\frac{\dot{H}}{H^2} < 1; \) slow roll: \( \epsilon \ll 1 \)]
Solving the coupled system of EoMs:

- Subtruncation of the consistent truncation:
  
  Can consistently set \( u \equiv 0, \ v \equiv 0, \ b \equiv 0 \)
  
  [EoMs identically solved]
  
  \( \rightarrow \) Study class of solutions with only nontrivial scalars:
  
  \( \phi(x^I), \ p(x^I), \ q(x^I) \)

- Look for quasi-de Sitter solutions (i.e. with \( H \approx const \)):
  
  In gauge/gravity duality context: these scalar fields - glueballs
  
  Discrete mass spectrum \( \rightarrow \) inflaton mass dynamically fixed
  
  \( \Rightarrow \) No \( \eta \) problem!
BUT:

Number of EoMs for scalar fields and metric functions is with one more than number of unknown functions

→ No solution?

Fortunately, we showed: [as long as $A'(r) \neq 0$]

One equation is dependent on the others!

Solutions with $H = \text{const}$:

• 3-parameter family with $q = -6p$ and $\phi = 0$
  [analytical solution]

• two 4-parameter families with $q = -\frac{3}{2}p$ and $\phi = 3p$
  [numerical solutions]
Solutions with time-dependent $H$:

Look for small time-dependent deviations from exact $H = \text{const}$ (i.e. pure $dS$) solution:  

\[ \text{[Recall: } -\frac{\dot{H}}{H^2} \ll 1 \text{]} \]

\[ \rightarrow \text{ Deform 5d metric ansatz and ansatz for scalar field(s)} \]

\[
\text{Metric: } \quad ds_5^2 = e^{2A} \left[-dt^2 + e^{2\tilde{H}} d\vec{x}^2\right] + dr^2
\]

- Expand in $\gamma \ll 1$ around the analytical solution:

\[
\phi = \gamma \phi_{(1)} + \gamma^3 \phi_{(3)} + \mathcal{O}(\gamma^5) \\
A = A_{(0)} + \gamma^2 A_{(2)} + \mathcal{O}(\gamma^4) \\
\tilde{H} = \tilde{H}_{(0)} + \gamma^2 \tilde{H}_{(2)} + \mathcal{O}(\gamma^4),
\]

where $A_{(0)}$, $\tilde{H}_{(0)} = H_0 t$ - an. sol.
Leading order solution:

At order $\gamma$: Single equation for $\phi_{(1)}$

At order $\gamma^2$: Coupled system for $A_{(2)}$, $\tilde{H}_{(2)}$, incl. $\phi_{(1)}^2$ terms

→ Obtain explicit solutions: $H(t)$ and $\phi(t)$

Recall that inflationary slow roll parameters:

$$\varepsilon = -\frac{\dot{H}}{H^2} \quad \text{and} \quad \eta = -\frac{\ddot{\phi}}{H \dot{\phi}}$$

For our solution:

$$\varepsilon \sim O(\gamma^2) \ll 1 \quad , \quad \eta = 3 + O(\gamma^2) \approx 3$$
Ultra-slow roll inflation:

For inflationary parameters $\varepsilon \ll 1$ and $\eta = 3$:

→ **Ultra-slow roll inflation**  [arXiv:gr-qc/0503017, W. Kinney]

Gives $n_s \approx 1$ (i.e. scale-invariant spectrum), but does not last for more than a few e-foldings

⇒ Can only be a transient phase, before usual slow roll

However, such a phase could explain **low-$l$ anomaly in CMB power spectrum**

$[ l \lesssim 40 \rightarrow \text{power deficit (\sim 10\%) compared to slow roll expectation}]$

( lower $l$ – larger angular scales )
Consider higher order in $\gamma$:

$$\phi = \gamma \phi^{(1)} + \gamma^3 \phi^{(3)} + O(\gamma^5)$$

$$A = A^{(0)} + \gamma^2 A^{(2)} + \gamma^4 A^{(4)} + O(\gamma^6)$$

$$\tilde{H} = \tilde{H}^{(0)} + \gamma^2 \tilde{H}^{(2)} + \gamma^4 \tilde{H}^{(4)} + O(\gamma^6)$$

Take: $\phi^{(1)} = 0$, $A^{(2)} = 0$, $H^{(2)} = 0$

(Satisfies the EoMs to order $\gamma^2$.)

So to leading order in $\gamma$: $\phi \sim O(\gamma^3) \rightarrow \phi^2 \sim O(\gamma^6)$

$\Rightarrow$ EoMs for $A^{(4)}$, $\tilde{H}^{(4)}$ decouple from $\phi^{(3)}$!

(Ultra-slow roll solution above: $A^{(2)}$, $\tilde{H}^{(2)}$, $\phi^{(1)} \sim O(\gamma^2)$.)

Toward slow roll inflation: (arXiv:1611.00295 [hep-th])
Toward slow roll inflation:

→ Solutions for $A(4)$, $\tilde{H}(4)$ are independent of $\phi(3)$ and vice versa.

In particular, integration constants in $\phi(3)$ are unrelated to those in $A(4)$, $\tilde{H}(4)$.

This relaxes constraint that led to ultra-slow roll at $O(\gamma^2)$.

⇒ Can show that, for suitable choices of the integration constants, one can obtain:

$$\eta \ll 1$$

→ Slow roll regime!
Summary

Found so far:

- Three multi-parameter solutions in 5d consistent truncation of IIB supergravity
  \[
  dS_4 \text{ space fibered over the fifth dimension}
  \]
- Ultra-slow roll glueball inflation model \([t\text{-dep. deformation}]\)
- Showed that it is possible to obtain slow roll regime

Open issues:

- Slow roll Glueball Inflation?...
- Microscopic realization?...
- Inflaton mass (mass-spectrum of fluctuations)?...
Thank you!